Circles

Introduction to Circles

Circles

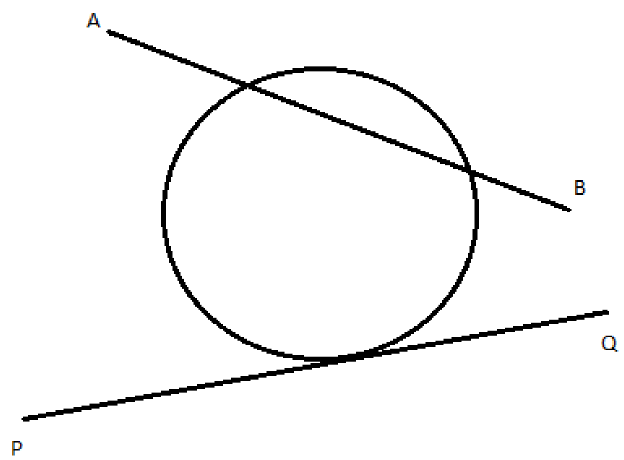
- The **set of all the points** in a plane that is at a **fixed distance** from a **fixed point** makes a circle.
- A **Fixed point** from which the set of points are at fixed distance is called **centre** of the circle.
- A circle divides the plane into 3 parts: **interior** (inside the circle), the **circle** itself and **exterior** (outside the circle)

Radius

- The **distance** between the **center** of the circle and any **point on its edge** is called the **radius.**

Tangent and Secant

A **line** that **touches** the circle at **exactly one point** is called its **tangent**.
A **line** that **cuts** a circle at **two points** is called as a **secant.**



In the above figure: PQ is the tangent and AB is the secant.

Chord

-The **line segment** within the circle joining any 2 points on the circle is called the chord.

Diameter

- A **Chord** passing through the center of the circle is called the **diameter.**
- The **Diameter is 2 times the radius** and it is the **longest chord**.

Arc

- The **portion** of a circle(curve) **between 2 points** is called an **arc**.
- Among the two pieces made by an arc, the **longer** one is called **major arc** and the **shorter** one is called **minor arc.**

Circumference

The **perimeter** of a circle is the **distance** covered by going around its **boundary once**. The perimeter of a circle has a special name: **Circumference**, which is π times the diameter which is given by the formula $2\pi r$

Segment and Sector

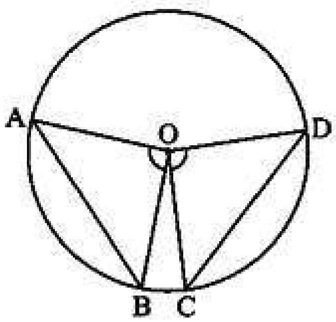
- A circular **segment** is a region of a circle which is "**cut off**" from the rest of the circle by a secant or a chord.
- **Smaller region** cut off by a chord is called **minor segment** and the **bigger region** is called **major segment**.

- A **sector** is the portion of a circle **enclosed by two radii and an arc**, where the **smaller area** is known as the **minor sector** and the **larger** being the **major sector**.
- For **2 equal** arcs or for semicircles - both the segment and sector is called the **semicircular region.**

Circles and Their Chords

Theorem of equal chords subtending angles at the center.

- Equal **chords** subtend equal **angles at the center**.

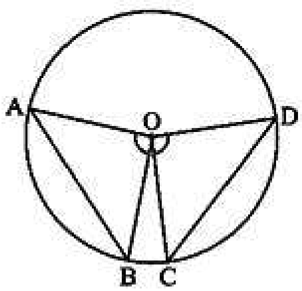


Proof : AB and CD are the 2 equal chords.
In $\Delta AOB \ and \ \Delta COD$

$OB = OC$ [Radii]
$OA = OD$ [Radii]
$AB = CD$ [Given]
$\Delta AOB \cong \Delta COD$ (SSS rule)
Hence, $\angle AOB = \angle COD$ [CPCT]

Theorem of equal angles subtended by different chords.

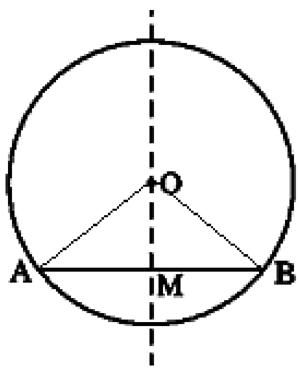
- If the **angles** subtended by the chords of a circle at the center are **equal**, then the **chords are equal.**



Proof : In $\Delta AOB \ and \ \Delta COD$
$OB = OC$ [Radii]
$\angle AOB = \angle COD$ [Given]
$OA = OD$ [Radii]
$\Delta AOB \cong \Delta COD$(SAS rule)
Hence, $AB = CD$ [CPCT]

Perpendicular from the center to a chord bisects the chord.

Perpendicular from the **center** of a circle to a **chord bisects the chord**.

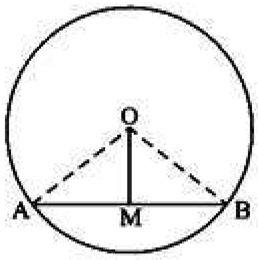


Proof: AB is a chord and OM is the perpendicular drawn from the center.
From $\Delta \ OMB \ and \ \Delta OMA$,
$\angle OMA = \angle OMB = 90^0$
$OA = OB$ (radii)
$OM = OM$ (common)

Hence, $\Delta OMB \cong \Delta OMA$ (RHS rule)
Therefore $AM = MB$ [CPCT]

A Line through the center that bisects the chord is perpendicular to the chord.

- A **line** drawn through the center of a circle to **bisect** a chord, is **perpendicular** to the chord.



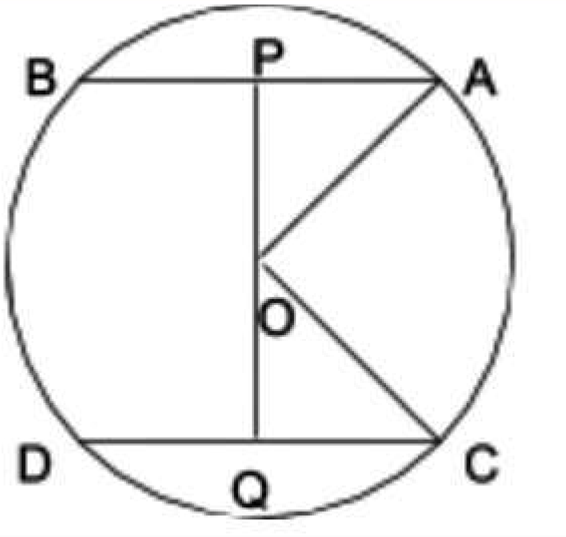
Proof: OM drawn from center to bisect chord AB .
From $\Delta\ OMA\ and\ \Delta OMB$,
$OA = OB$ (Radii)
$OM = OM$ (common)
$AM = BM$ (Given)
Therefore, $\Delta OMA \cong \Delta OMB$ (SSS rule)
$\Rightarrow \angle OMA = \angle OMB$ (C.P.C.T)
But, $\angle OMA + \angle OMB = 180^0$
Hence, $\angle OMA = \angle OMB = 90^0$
$\Rightarrow OM \perp AB$

Circle through 3 points

- There is **one** and **only** one **circle** passing through **three given noncollinear points.**
- A unique circle passes through 3 vertices of a triangle ABC called as the **circumcircle.** The **centre** and **radius** are called the **circumcenter** and **circumradius** of this triangle, respectively.

Equal chords are at equal distances from the center.

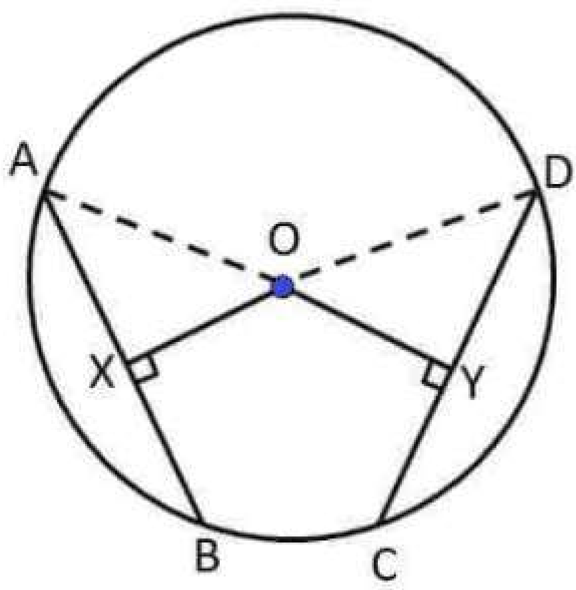
Equal chords of a circle(or of congruent circles) are **equidistant from the centre** (or centres).



Proof : Given, $AB = CD$, O is the centre. Join OA and OC.
Draw, $OP \perp AB, OQ \perp CD$
In $\Delta OAP\ and \Delta OCQ$,
$OA = OC$ (Radii)
$AP = CQ$ (AB = CD $\Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$, since OP and OQ bisects the chords AB and CD.)
$\Delta OAP \cong \Delta OCQ$ (RHS rule)
Hence, $OP = OQ$ (C.P.C.T.C)

Chords equidistant from center are equal

Chords **equidistant** from the center of a circle are **equal in length.**



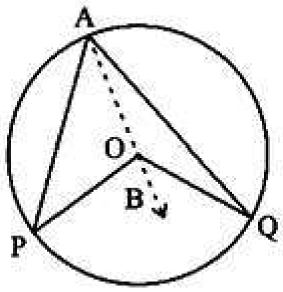
Proof : Given OX = OY (The chords AB and CD are at equidistant)
$OX \perp AB, OY \perp CD$
In $\Delta AOX\ and\ \Delta DOY$
$\angle OXA = \angle OYD$ (Both 90^0)

$OA = OD$ (Radii)
$OX = OY$ (Given)
$\Delta AOX \cong \Delta DOY$ (RHS rule)
Therefore $AX = DY$ (CPCT)
Similarly $XB = YC$
So, $AB = CD$

Circles and Quadrilaterals

Angle subtended by an arc of a circle on the circle and at the center

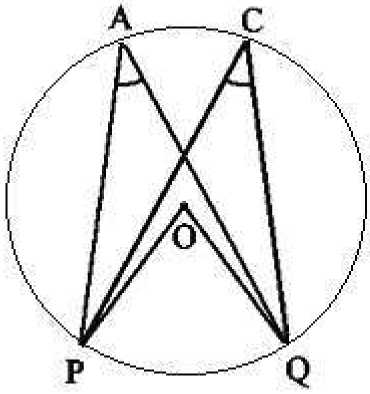
The **angle** subtended by **an arc** at the **centre is double** the angle subtended by it on any **part of the circle**.



Here PQ is the arc of a circle with centre O, that subtends $\angle POQ$ at the centre.
Join AO and extend it to B.
In ΔOAQ
$OA = OQ$..... [Radii]
Hence, $\angle OAQ = \angle OQA$....[Property of isosceles triangle]
Implies $\angle BOQ = 2\angle OAQ$ .....[Exterior angle of triangle = Sum of 2 interior angles]
Similarly, $\angle BOP = 2\angle OAP$
$\Rightarrow \angle BOQ + \angle BOP = 2\angle OAQ + 2\angle OAP$
$\Rightarrow \angle POQ = 2\angle PAQ$
Hence proved

Angles in same segment of a circle.

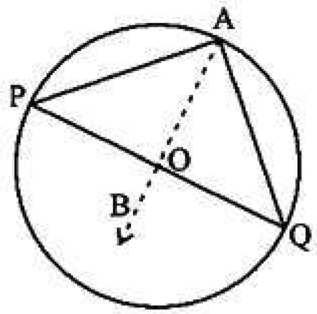
-**Angles** in the **same segment** of a circle are **equal.**



Consider a circle with centre O.
$\angle PAQ$ and $\angle PCQ$ are the angles formed in the major segment PACQ with respect to the arc PQ.
Join OP and OQ
$\angle POQ = 2\angle PAQ = 2\angle PCQ$ .....[Angle subtended by an arc at the centre is double the angle subtended by it in any part of the circle]
$\Rightarrow \angle PCQ = \angle PAQ$
Hence proved

Angle subtended by diameter on the circle

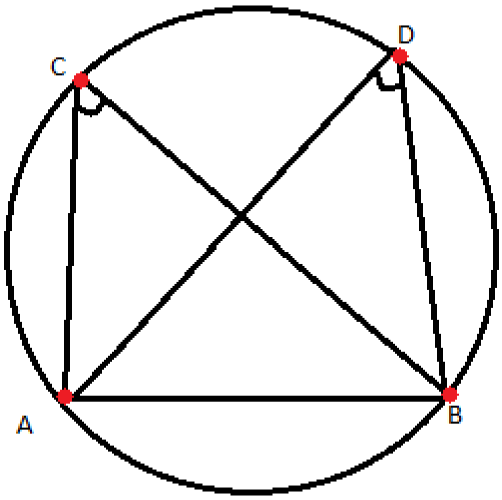
- **Angle** subtended by **diameter** on a circle is a **right angle**.(Angle in a semicircle is a right angle)



Consider a circle with center O, POQ is the diameter of the circle.
$\angle PAQ$ is the angle subtended by diameter PQ at the circuference.
$\angle POQ$ is the angle subtended by diameter PQ at the center.
$\angle PAQ = \frac{1}{2}\angle POQ$... [Angle subtended by arc at the centre is double the angle at any other part]
$\angle PAQ = \frac{1}{2} \times 180^0 = 90^0$
Hence proved

Line segment that subtends equal angles at two other points

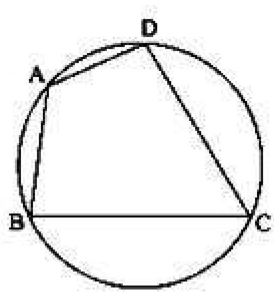
- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.(i.e they are concyclic)



Here $\angle ACB = \angle ADB$ and all 4 points A,B,C,D are concyclic.

Cyclic Quadrilateral

- A **Quadrilateral** is called a **cyclic quadrilateral** if all the **four vertices lie on a circle**.



In a circle, if all **four points** A, B, C and D lie **on the circle**, then quadrilateral ABCD is a **cyclic quadrilateral.**

Sum of opposite angles of a cyclic quadrilateral

- If sum of a pair of opposite angles of a quadrilateral is 180 degree, the quadrilateral is cyclic.

Sum of pair of opposite angles in quadrilateral

- The sum of either pair of opposite angles of a cyclic quadrilateral is 180 degree