READING MATERIAL

CLASS IX CHAPTER 9 FORCE AND LAWS OF MOTION

Inertia and Mass

- First Law
- Inertia and Mass
- State of Motion
- Balanced vs. Unbalanced Forces

Newton's first law of motion states that "An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force." Objects tend to "keep on doing what they're doing." In fact, it is the natural tendency of objects to resist changes in their state of motion. This tendency to resist changes in their state of motion is described as **inertia**.

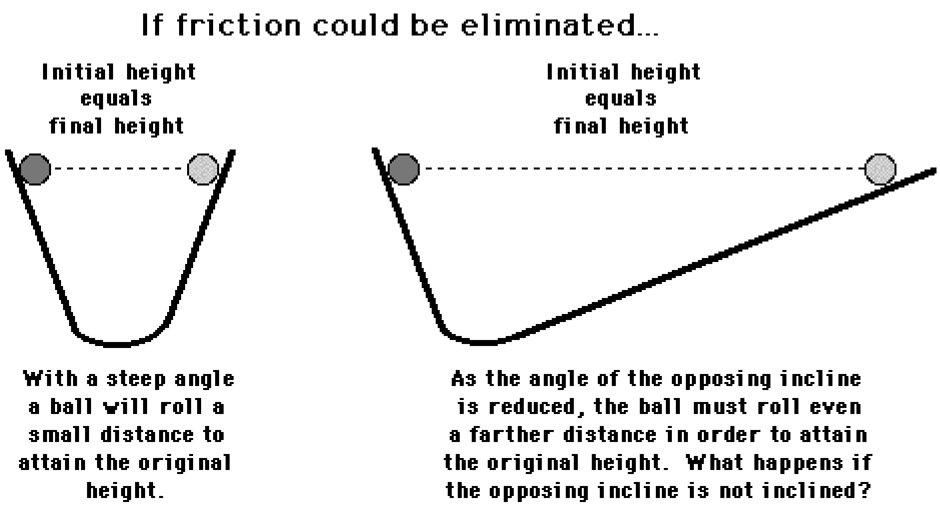
Inertia: the resistance an object has to a change in its state of motion.

Newton's conception of inertia stood in direct opposition to more popular conceptions about motion. The dominant thought prior to Newton's day was that it was the natural tendency of objects to come to a rest position. Moving objects, so it was believed, would eventually stop moving; a force was necessary to keep an object moving. But if left to itself, a moving object would eventually come to rest and an object at rest would stay at rest; thus, the idea that dominated people's thinking for nearly 2000 years prior to Newton was that it was the natural tendency of all objects to assume a rest position.

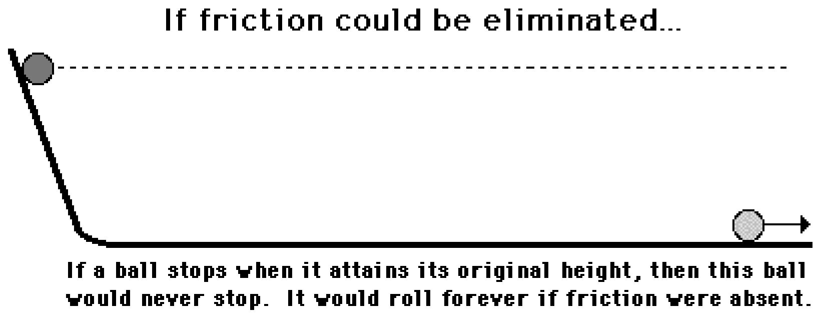
Galileo and the Concept of Inertia

Galileo, a premier scientist in the seventeenth century, developed the concept of inertia. Galileo reasoned that moving objects eventually stop because of a force called friction. In experiments using a pair of inclined planes facing each other, Galileo observed that a ball would roll down one plane and up the opposite plane to approximately the same height. If smoother planes were used, the ball would roll up the opposite plane even closer to the original height. Galileo reasoned that any difference between initial and final heights was due to the presence of friction. Galileo postulated that if friction could be entirely eliminated, then the ball would reach exactly the same height.

Galileo further observed that regardless of the angle at which the planes were oriented, the final height was almost always equal to the initial height. If the slope of the opposite incline were reduced, then the ball would roll a further distance in order to reach that original height.



Galileo's reasoning continued - if the opposite incline were elevated at nearly a 0-degree angle, then the ball would roll almost forever in an effort to reach the original height. And if the opposing incline was not even inclined at all (that is, if it were oriented along the horizontal), then ... an object in motion would continue in motion... .



NUMERICALS

Question 1: Calculate the force needed to speed up a car with a rate of $5ms^{-2}$, if the mass of the car is 1000 kg.

Solution: According to question:

Acceleration (a) =5m/s2=5m/s2 and Mass (m) = 1000 kg, therefore, Force (F) =?
We know that, F=m×aF=m×a
=1000kg×5m/s2=1000kg×5m/s2
=5000kg m/s2=5000kg m/s2
Therefore, required Force =5000m/s2=5000m/s2 or 5000 N

Question 2: If the mass of a moving object is 50 kg, what force will be required to speed up the object at a rate of $2ms^{-2}$?

Solution: According to the question;

Acceleration (a) =2ms−2=2ms-2 and Mass (m) = 50 kg, therefore, Force (F) =?
We know that, F=m×aF=m×a
=50kg×2m/s2=50kg×2m/s2

=100kg m/s2=100kg m/s2
Therefore, required Force =100m/s2=100m/s2 or 100 N

Question 3: To accelerate an object to a rate of 2m/s^2, 10 N force is required. Find the mass of object.

Solution: According to the question:

Acceleration (a) = 2m/s^2, Force (F) = 10N, therefore, Mass (m) = ?
We know that, F=m×aF=m×a
⇒10N=m×2m/s2⇒10N=m×2m/s2
⇒m=102 kg=5 kg⇒m=102 kg=5 kg
Thus, the mass of the object = 5 kg

Question 4: An object gets 50 second to increase the speed from 10m/s to 50m/s. If the mass of the object is 1000 kg, what force will be required to do so?

Solution: According to the question:

Initial velocity (u) = 10m/s, final velocity (v) = 50m/s, time (t) = 50 second, Mass (m) = 1000 kg,
Therefore, force (F)=?
We know that, Force (F) =mv−ut=mv-ut
∴F=1000 kg 50 m/s−10 m/s50 s∴F=1000 kg 50 m/s-10 m/s50 s
⇒F=1000 kg×4050 ms−2⇒F=1000 kg×4050 ms-2
⇒F=20 kg×40ms−2⇒F=20 kg×40ms-2
⇒F=800 kg ms−2=800N⇒F=800 kg ms-2=800N
Thus required force = 800 N

Question 5: A vehicle having mass equal to 1000 kg is running with a speed of 5m/s. After applying the force of 1000N for 10 second what will be the speed of vehicle?

Solution: According to the question:

Mass of (m) = 1000 kg, Force, (F) = 1000 N, time (t) = 10s, Initial velocity (u) = 5m/s

Therefore, Final velocity (v) =?

We know that, Force (F) =mv−ut=mv-ut

∴1000N=1000 kg v−5m/s10s∴1000N=1000 kg v-5m/s10s

⇒ 1000 kg m/s^2 × 10s = 1000 kg (v – 5m/s)

⇒ 10000 kg m/s = 1000 kg × v – 5000 kg m/s

⇒ 10000 kg m/s + 5000kg m/s = 1000kg × v

⇒ 15000 kgm/s = 1000 kg × v

⇒v=15000 kg m/s1000 kg=15 m/s⇒v=15000 kg m/s1000 kg=15 m/s

Thus, the velocity of the vehicle will be 15m/s.

Newton’s laws of Motion

Foundations of Newtonian Mechanics

Three fundamental quantities:

(i) Mass,

(ii) Motion &

(iii) Force

Excerpts from Newton’s Principia (Book 1)

Mass

The quantity of matter is the measure of the same arising from it’s density and bulk conjointly.

Motion

The quantity of motion is the measure of the same arising from the velocity and quantity of matter conjointly.

Force (Definiton # 1)

The *vis insita:* an innate forces of matter, is a power of resisting, by which every body, as much as in it lies, continues in its present state, whether it be of rest, or of moving uniformly forward in a right line.”

Force (Definition # 2)

An impressed force is an action exerted upon a body, in order to change its state, either of rest, or of moving uniformly forward in a right line.

These definitions gave rise to the famous three laws: known as Newton’s laws of motion.

Law 1

Every body continues in it’s state of rest or of uniform rectilinear motion except if it is compelled by forces acting on it to change that sate.

Law 2

The change of motion is proportional to the applied force and takes place in the direction of the straight line along which that force acts.

Laws 3

To every action there is always an equal and contrary reaction; or the mutual actions of any two bodies are always equal and oppositely directed along the same straight line.

By solving Newton's laws we shall find $\boldsymbol{r}(t)$.

$r(t) = 0$: implies that the body is in rest for all time.

In general,

$$r(t)=(x(t), y(t), z(t)) \text{ or } (r(t), \theta(t), \phi(t))$$

Example

$$r(t) = (v_x t + x_0;\ 0;\ v_z t + z_0 - gt^2 / 2)$$

represents uniform motion in the x-direction with v_x as the velocity, in a state of rest in the y-direction and is having a uniform velocity v_z and a free fall in the gravitational field.

Mechanics of particles

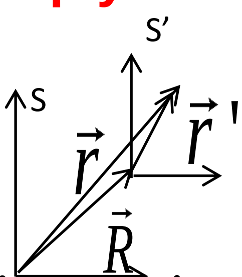
Classical Mechanics	Non-relativistic (Newton's Laws)	Relativistic (Special Theory of Relativity)
Quantum Mechanics	Non-Relativistic (Schroedinger Equation)	Relativistic (Dirac Equation)

Newton’s first law of motion

1. **Gives a definition of (zero) force**
2. **Defines an *inertial frame*.**

Zero Force: When a body moves with *constant velocity in a straight line,* either there are no forces present or the net force acting on the body is zero $\sum_i \vec{F}_i = 0$ If the body changes its velocity, then there must be an acceleration, and hence a total non-zero force must be present. Velocity can change due to change in its magnitude or due to change in its direction or change in both.

Inertial frame: If the relative velocity between the two reference frames is constant, then the relative acceleration between the two reference frames is zero, $\vec{\mathbf{A}} = \frac{d\vec{\mathbf{V}}}{dt} = \vec{\mathbf{0}}$ and the reference frames are considered to be *inertial reference frames.* The inertial frame is then simply a frame of reference in which the first law holds.



$$\vec{r}\,' = \vec{r} - \vec{v}t, \quad \vec{v} = \frac{d\vec{R}}{dt}$$

Galilean transformation

Is Earth an inertial frame?

The first law does *not hold in an arbitrary frame. For example, it fails in the frame of a rotating* turntable.

Newton’s Second law of motion:

If any force generates a change in motion, a double force will generate double change in the motion, a triple force will correspond to triple change in the motion, whether that force is impressed altogether and at once or gradually or successively.

Change of motion is described by the change in momentum of body. For a point mass particle, the momentum is defined as $\vec{p} = m\vec{v}$

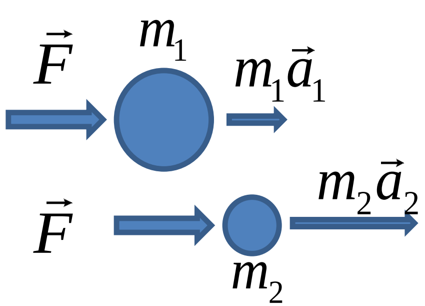
Suppose that a force is applied to a body for a time interval Δt . *The impressed force or impulse produces a change in the momentum of the body,*

$$\bar{\vec{\mathbf{I}}} = \bar{\vec{\mathbf{F}}}\,\Delta t = \Delta\vec{\mathbf{p}}$$

The instantaneous action of the total force acting on a body at a time *t is defined by taking the mathematical limit as the time interval Δt becomes smaller and smaller,*

$$\vec{\mathbf{F}}^{\text{total}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{p}}}{\Delta t} \equiv \frac{d\vec{\mathbf{p}}}{dt}$$

$$\vec{F}^{\text{total}} = \frac{d}{dt}\left(m\vec{v}\right) = m\frac{d\vec{v}}{dt}$$



$$\frac{m_1}{m_2} = \frac{a_2}{a_1}$$

Inertial mass

$$\vec{\mathbf{F}}^{\text{total}} = m\vec{\mathbf{a}}$$

Inertial mass $\equiv$ Gravitational mass

Newton’s third law of motion:

Consider two bodies engaged in a mutual interaction. Label the bodies 1 and 2 respectively. Let $\vec{F}_{1,2}$ be the force on body 1 due to the interaction with body 2, and $\vec{F}_{2,1}$ be the force on body 2 due to the interaction with body 1.

$$\vec{\mathbf{F}}_{1,2} = -\vec{\mathbf{F}}_{2,1}$$

$$\text{Gravitational force: } \vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12} \qquad \hat{r}_{12} = -\hat{r}_{21}$$

$$\text{Coulomb force: } \vec{F}_{12} = k\frac{q_1 q_2}{r^2}\hat{r}_{12} \qquad \vec{F}_{12} = -\vec{F}_{21}$$

All real Forces arise due to interaction!

If the acceleration of a body is the result of an outside force, then somewhere in the universe there must be an equal and opposite force acting on another body. The interaction may be a complicated one, but as long as the forces are equal and opposite, Newton's laws are satisfied.

Newton’s 3rd law emphasizes Conservation of Momentum

Validity of Newton's laws

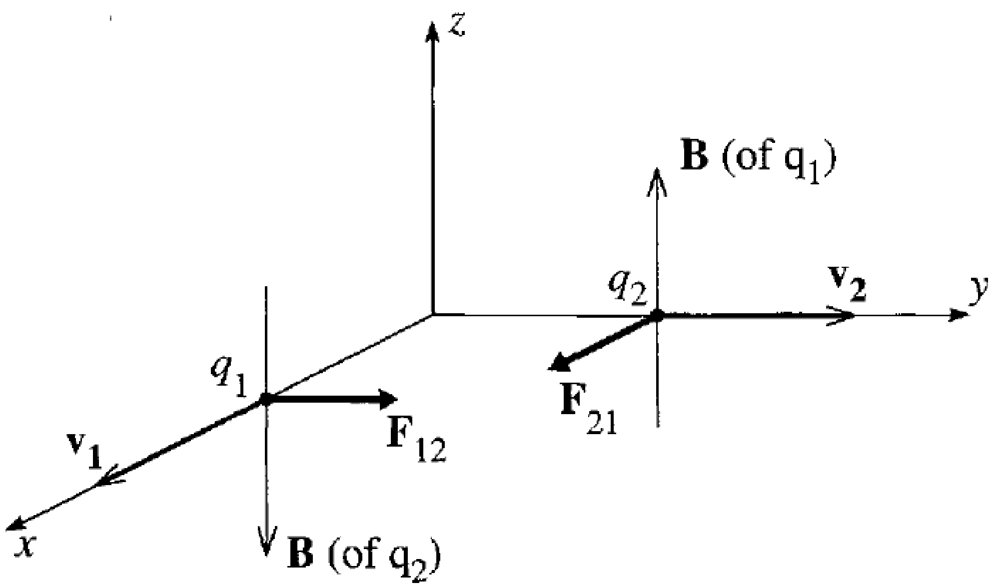
- **Validity of the first two laws**

  - The first law is always valid (add a pseudo force).

  - The second law $\mathbf{F} = \dot{p}$ holds but $\mathbf{F}$ and p have different expressions in the relativistic limit.

- **The 3rd law is not valid in the relativistic limit. Why????**

Consider two positive charges



Each of the positive charges q_1 and q_2 produces a magnetic field that exerts a force on the other charge. The resulting magnetic forces $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ do not obey Newton's third law.

Momentum conservation is not valid

Application of Newton's laws: Prescription

Step 1: Divide a composite system into constituent systems each of which can be treated as a point mass.

Step 2: Draw free body force diagrams for each point mass.

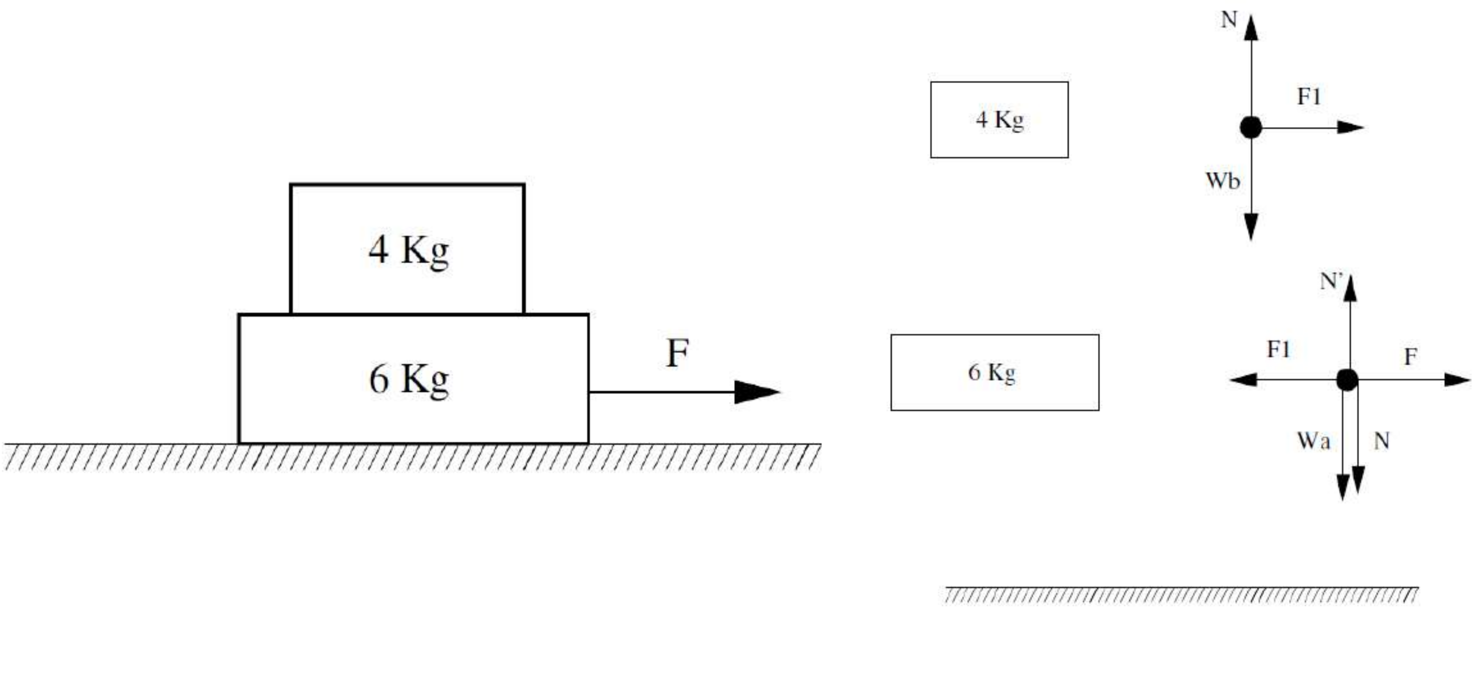
Step 3: Introduce a coordinate system, the inertial frame, and write the equations of motion.

Step 4: Motion of a body may be constrained to move along certain path or plane. Express each constraint by an equation called constraint equation.

Step 6: Identify the number of unknown quantities. There must be enough number of equations (Equations of motion + constraint equations) to solve for all the unknown quantities.

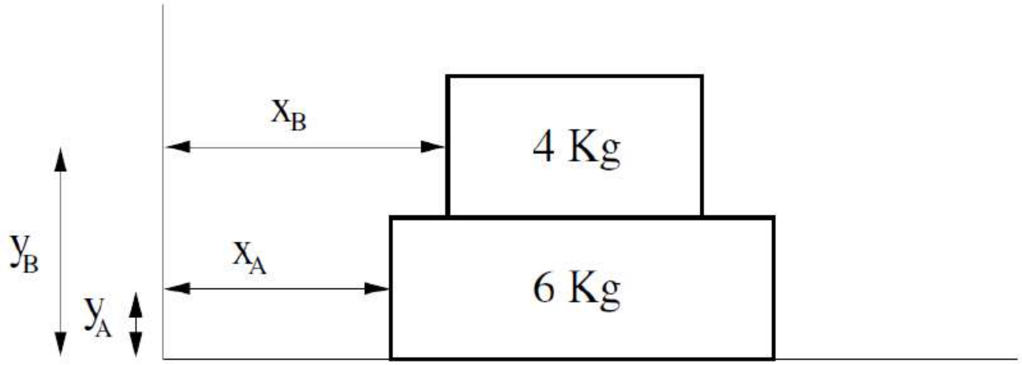
Example 1

A 4 Kg block rests on top of a 6 Kg block, which rests on a frictionless table. Coefficient of friction between blocks is 0.25. A force $F = 10N$ is applied to the lower block.



Identify the constraints

Fix the coordinate system to the table.



$$
\begin{aligned}
y_A &= const \\
y_B &= const \\
x_A &= x_B + const
\end{aligned}
$$

EOM in x and y-directions

Equations of Motion in Y direction.

$$
\begin{aligned}
m_A \ddot{y_A} &= N' - W_A - N \\
m_B \ddot{y_B} &= N - W_B
\end{aligned}
$$

Constraints

$$
\begin{aligned}
\ddot{y_A} &= 0 \\
\ddot{y_B} &= 0
\end{aligned}
$$

Solution

$$
\begin{aligned}
N' &= W_A + W_B \\
N &= W_B
\end{aligned}
$$

Equations of Motion in X direction.

$$
\begin{aligned}
m_A \ddot{x_A} &= F - F_1 \\
m_B \ddot{x_B} &= F_1
\end{aligned}
$$

Constraints

$$
\ddot{x_A} = \ddot{x_B}
$$

Solution

$$
\begin{aligned}
\ddot{x_A} = \ddot{x_B} &= \frac{F}{m_A + M_B} = 1\mathrm{m/s}^2 \\
F_1 &= m_B \ddot{x_B} = 4N
\end{aligned}
$$

The force $F_1 < \mu N = 10$ N, the maximum frictional force between the blocks. Hence the solution is consistent with assumption.

What would be the motion if $F = 40$ N?

Velocity and acceleration in cylindrical polar coordinates :

$$\vec{r} = \rho\cos\phi\hat{i} + \rho\sin\phi\hat{j} + z\hat{k} = \rho\hat{\rho} + z\hat{k}$$

$$\hat{\rho} = \cos\phi\hat{i} + \sin\phi\hat{j}$$

$$\hat{\phi} = -\sin\phi\hat{i} + \cos\phi\hat{j}$$

$$\hat{z} = \hat{k}$$

$$\dot{\hat{\rho}} = \frac{\partial\hat{\rho}}{\partial\rho}\dot{\rho} + \frac{\partial\hat{\rho}}{\partial\phi}\dot{\phi} + \frac{\partial\hat{\rho}}{\partial z}\dot{z} = \hat{\phi}\dot{\phi}$$

$$\dot{\hat{\phi}} = \frac{\partial\hat{\phi}}{\partial\rho}\dot{\rho} + \frac{\partial\hat{\phi}}{\partial\phi}\dot{\phi} + \frac{\partial\hat{\phi}}{\partial z}\dot{z} = -\hat{\rho}\dot{\phi}$$

$$\dot{\hat{z}} = \frac{\partial\hat{z}}{\partial\rho}\dot{\rho} + \frac{\partial\hat{z}}{\partial\phi}\dot{\phi} + \frac{\partial\hat{z}}{\partial z}\dot{z} = 0$$

$$\vec{v} = \dot{\vec{r}} = \dot{\hat{\rho}}\rho + \hat{\rho}\dot{\rho} + \dot{\hat{z}}z + \hat{z}\dot{z} = \hat{\rho}\dot{\rho} + \hat{\phi}\rho\dot{\phi} + \hat{z}\dot{z}$$

$$\boxed{\vec{v} = \hat{\rho}\dot{\rho} + \hat{\phi}\rho\dot{\phi} + \hat{z}\dot{z}}$$

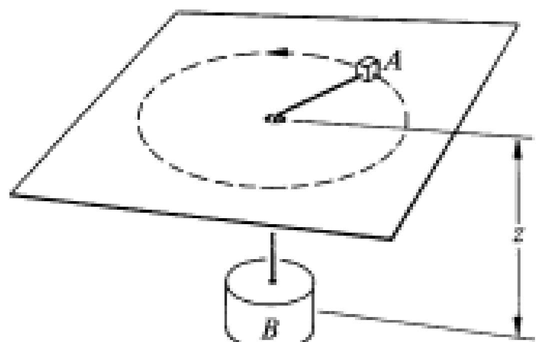
$$\vec{a} = \dot{\vec{v}} = \dot{\hat{\rho}}\dot{\rho} + \hat{\rho}\ddot{\rho} + \dot{\hat{\phi}}\rho\dot{\phi} + \hat{\phi}\dot{\rho}\dot{\phi} + \hat{\phi}\rho\ddot{\phi} + \dot{\hat{z}}\dot{z} + \hat{z}\ddot{z}$$

$$= \hat{\phi}\dot{\phi}\dot{\rho} + \hat{\rho}\ddot{\rho} - \hat{\rho}\rho\dot{\phi}^2 + \hat{\phi}\dot{\rho}\dot{\phi} + \hat{\phi}\rho\ddot{\phi} + \hat{z}\ddot{z}$$

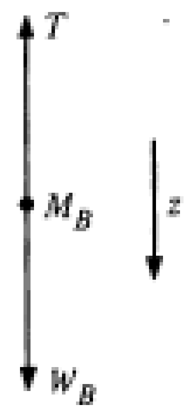
$$\boxed{\vec{a} = \hat{\rho}\left(\ddot{\rho} - \rho\dot{\phi}^2\right) + \hat{\phi}\left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right) + \hat{z}\ddot{z}}$$

Example 2

A horizontal frictionless table has a small hole in its center. Block A on the table is connected to block B hanging beneath by a string of negligible mass which passes through the hole. Initially, B is held stationary and A rotates at constant radius r_o with steady angular velocity ω_0. If B is released at t = 0, what is its acceleration immediately afterward?



Two movable bodies and their free body diagrams:



Cylindrical polar coordinate system.

Equations of motion:

$$-T = M_A(\ddot{r} - r\dot{\theta}^2) \qquad \text{Radial}$$
$$0 = M_A(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \qquad \text{Tangential}$$
$$W_B - T = M_B\ddot{z} \qquad \text{Vertical.}$$

Constraint equations: $r + z = l. \longrightarrow \ddot{r} = -\ddot{z}.$

Unknowns: $a_r, a_\theta, \ddot{z}, T$ Four unknowns and four equations.

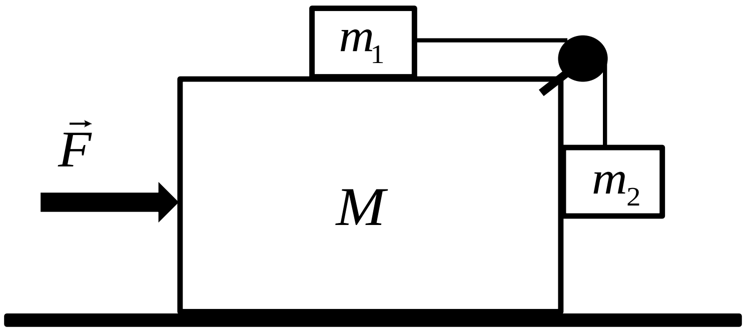
$$\ddot{z} = \frac{W_B - M_A r\dot{\theta}^2}{M_A + M_B}.$$

Immediately after B is released $r = r_0$ and $\dot{\theta} = \omega_0$

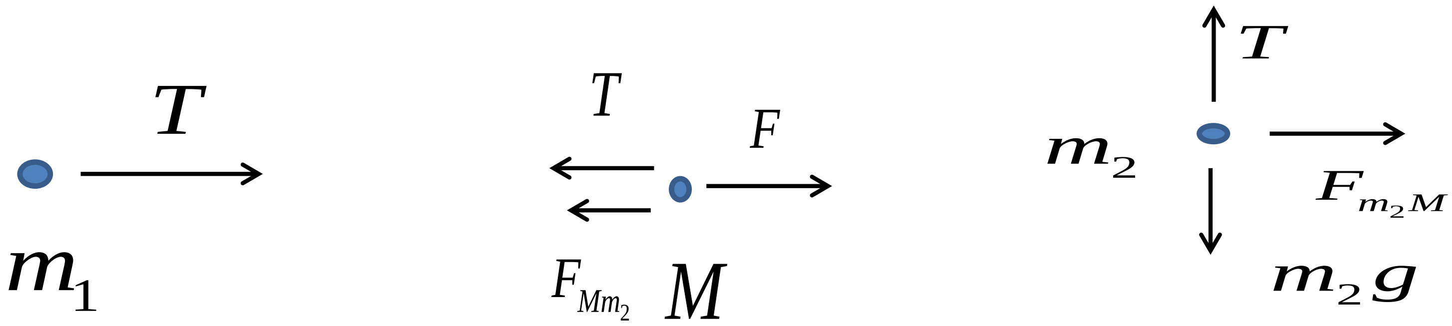
$$\ddot{z}(0) = \frac{W_B - M_A r_0 \omega_0{}^2}{M_A + M_B}.$$

Example 3

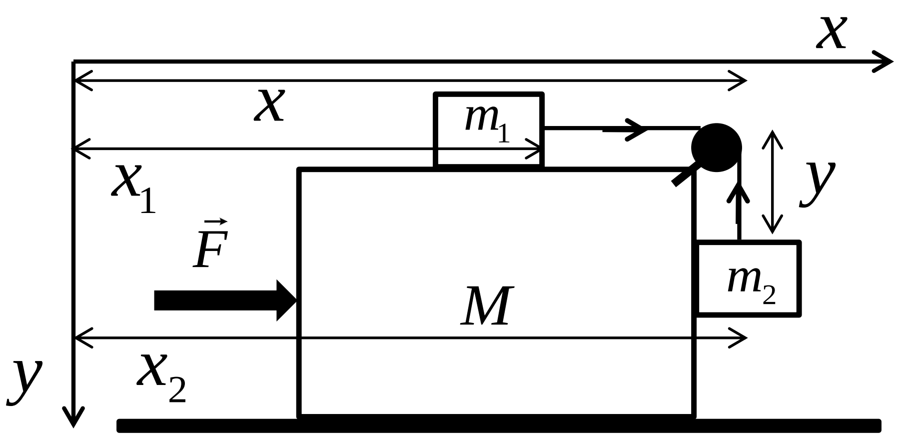
Determine the acceleration vectors of all mobile bodies in the following situation which are always in contact. All surfaces are frictionless, pulley is massless, and the string of length *L* is massless and inextensible.



Free body diagrams:



Specification of coordinates:



Equations of motion:

$$m_1 a_{1x} = T \qquad F - T - F_{Mm_2} = Ma_x$$

$$F_{m_2M} = m_2 a_{2x}$$

$$m_2 g - T = m_2 a_{2y}$$

3rd law of motion: $$F_{Mm_2} = F_{m_2M}$$

Constraint equations:

$$x = x_2 \quad \Rightarrow \quad a_x = a_{2x}$$

$$x - x_1 + y = L \quad \Rightarrow \quad a_x - a_{1x} + a_{2y} = 0$$

Unknowns: $T, F_1, F_2, a_x, a_{1x}, a_{2x}, a_{2y}$

There are seven equations and seven unknowns.

Accelerations:

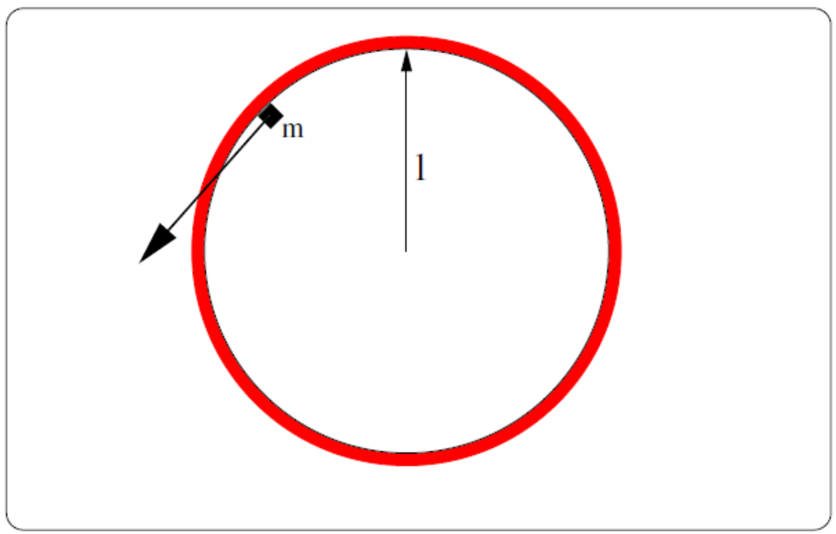
$$a_{1x} = \frac{m_2 \left[F + \left(M + m_2 \right) g \right]}{\left(M + m_2 \right)\left(m_1 + m_2 \right) + m_1 m_2}$$

$$a_x = a_{2x} = \frac{F \left(m_1 + m_2 \right) - m_1 m_2 g}{\left(M + m_2 \right)\left(m_1 + m_2 \right) + m_1 m_2}$$

$$a_{2y} = \frac{m_2 \left(M + m_1 + m_2 \right) g - m_2 F}{\left(M + m_2 \right)\left(m_1 + m_2 \right) + m_1 m_2}$$

Example 4

A block of mass m slides on a frictionless table. It is constrained to move move inside a ring of radius l fixed to the table. At $t = 0$ the block is touching the ring and has a velocity v_0 in tangential direction.



Find the velocity of the mass at subsequent times.

Constraint Equation is $r = l$, that is $\dot{r} = \ddot{r} = 0$.

Equations of Motion

$$
\begin{aligned}
m\left(\ddot{r} - r\dot{\theta}^2\right) = -ml\dot{\theta}^2 &= -N \\
m\left(r\ddot{\theta} - 2\dot{r}\dot{\theta}\right) = mr\ddot{\theta} &= -f
\end{aligned}
$$

Eliminating N, we get

$$
\begin{aligned}
\ddot{\theta} &= -\mu\dot{\theta}^2 \\
v(t) &= l\dot{\theta} \\
&= \frac{v_0}{1 + \mu v_0 t/l}
\end{aligned}
$$