POLYNOMIAL

POLYNOMIAL

- **Polynomial** comes from *poly-* (meaning "many") and *-nomial* (in this case meaning "term") ... so it says "many terms".
- A polynomial can have:
- **constants** (like **3**, **-20**, or **½**)
- **variables** (like ***x*** and ***y***)
- **exponents** (like the 2 in y^2), but only **0, 1, 2, 3, ...** etc are allowed
- that can be combined using **addition, subtraction, multiplication and division** ...

... except ...

- ... **not** division by a variable (so something like **2/x** is right out).

- So: A polynomial can have constants, variables and exponents, but never division by a variable.

Polynomials

A polynomial is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0$$

where the $a_n, a_{n-1}, \ldots a_1, a_0$

are real numbers and *n* is a nonnegative integer.

The **domain** of a **polynomial** function is **the set of real numbers**

Polynomials

An expression containing variables, constant and any arithematic operation is called polynomial.

Polynomial comes from *poly-* (meaning "many") and *-nomial* (in this case meaning "term") ... so it says "many terms"

$4xy^2 + 3x - 5$

terms

Coefficient Variable

$4x - 7 = 5$

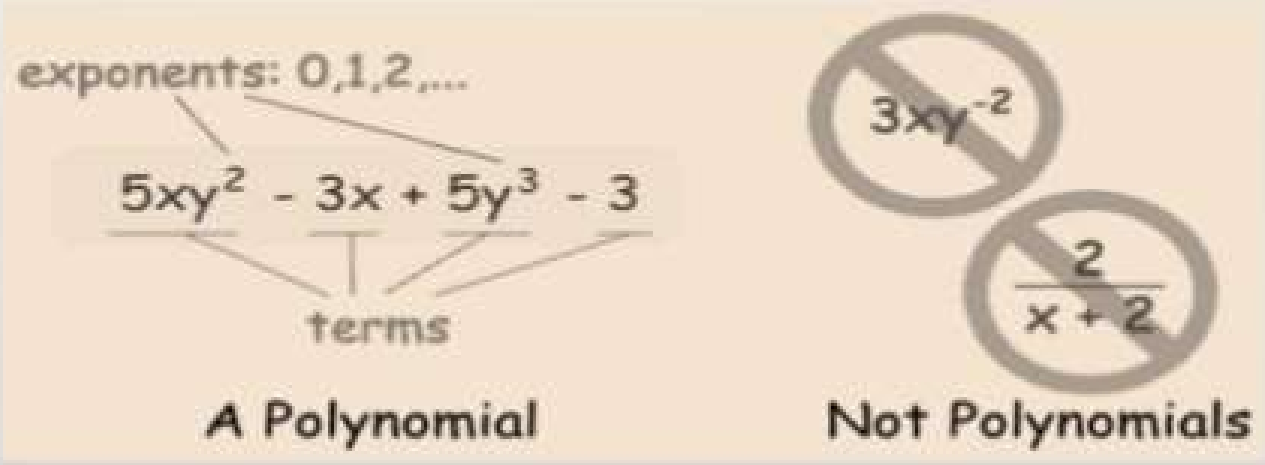
Operator Constants

exponents: 0,1,2,...

$5xy^2 - 3x + 5y^3 - 3$

WHAT IS A POLYNOMIAL

- A polynomial is an expression made with constants, variables and exponents, which are combined using addition, substraction and mutiplication but not division.
- The exponents can only be 0,1,2,3.... etc.
- A polynomial cannot have infinite number of terms.



$x^2 + 2x - 3$

Polynomial

$x^2 + 3x^{\frac{3}{5}} - \frac{2}{x}$

Not a Polynomial

– Polynomials and Polynomial Functions

Definitions

Term: a number or a product of a number and variables raised to a power.

$$3, \quad 5x^2, \quad -2x, \quad 9x^2y$$

Coefficient: the numerical factor of each term.

$$5x^2, \quad -2x, \quad 9x^2y$$

Constant: the term without a variable.

$$3, \quad -6, \quad 5, \quad 32$$

Polynomial: a finite sum of terms of the form ax^n, where a is a real number and n is a whole number.

$$-15x^3 + 2x^2 - 5 \qquad 21y^6 - 7y^5 - 2y^3 + 6y$$

Polynomials and Polynomial Functions

Definitions

Monomial: a polynomial with exactly one term.

$$ax^2, \quad rt, \quad 2x^4, \quad -9m, \quad 9x^2y$$

Binomial: a polynomial with exactly two terms.

$$x-8, \quad r-3, \quad 5x^2+2x, \quad -2x+9x^2y$$

Trinomial: a polynomial with exactly three terms.

$$x^2+x-8, \quad r^5+3r-3, \quad 5x^2+2x-7$$

Vocabulary

Monomial: A number, a variable or the product of a number and one or more variables.

Polynomial: A monomial or a sum of monomials.

Binomial: A polynomial with exactly two terms.

Trinomial: A polynomial with exactly three terms.

Coefficient: A numerical factor in a term of an algebraic expression.

Vocabulary

- **<u>Monomials</u> - a number, a variable, or a product of a number and one or more variables.** $4x$, $20x^2yw^3$, -3, a^2b^3, and $3yz$ are all monomials.

- **<u>Polynomials</u> – one or more monomials added or subtracted**
  - $4x + 6x^2$, $20xy - 4$, and $3a^2 - 5a + 4$ are all polynomials.

Polynomials and Polynomial Functions

Definitions

The Degree of a *Term* with one variable is the exponent on the variable.

$$5x^2 \Rightarrow 2, \quad 2x^4 \Rightarrow 4, \quad -9m \Rightarrow 1$$

The Degree of a *Term* with more than one variable is the sum of the exponents on the variables.

$$-7x^2y \Rightarrow 3, \quad 2x^4y^2 \Rightarrow 6, \quad -9mn^5z^4 \Rightarrow 10$$

The Degree of a *Polynomial* is the greatest degree of the terms of the polynomial variables.

$$2x^3 - 3x + 7 \Rightarrow 3, \quad 2x^4y^2 + 5x^2y^3 - 6x \Rightarrow 6$$

Polynomials and Polynomial Functions

Practice Problems

Identify the degrees of each term and the degree of the polynomial.

$5x^3 - 4x^2 + 5x$

3 2 1

3

$4a^2b^4 + 3a^3b^5 - 9b^4 + 4$

6 8 3 0

8

$4x^5y^4 + 5x^4y^6 - 6x^3y^3 + 2xy$

9 10 6 2

10

Polynomials and Polynomial Functions

Practice Problems

Evaluate each polynomial function

$f(-1) \qquad f(x) = 3x^2 - 10$

$3(-1)^2 - 10 \Rightarrow 3 \cdot 1 - 10 \Rightarrow 3 - 10 \Rightarrow -7$

$g(3) \qquad g(y) = 6y^2 + 11y - 20$

$6(3)^2 + 11(3) - 20 \Rightarrow 6 \cdot 9 + 33 - 20 \Rightarrow$

$54 + 33 - 20 \Rightarrow 87 - 20 \Rightarrow 67$

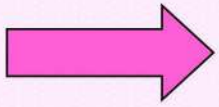
Adding Polynomials

Some people prefer to add polynomials by stacking them.
If you choose to do this, be sure to line up the like terms!

$(x^2 + 3x + 1) + (4x^2 + 5)$ 

$$\begin{array}{r} (x^2 + 3x + 1) \\ +\ (4x^2 \qquad +5) \\ \hline \boxed{\mathbf{5x^2 + 3x + 6}} \end{array}$$

Stack and add these polynomials: $(2a^2+3ab+4b^2) + (7a2+ab+-2b^2)$

$(2a^2+3ab+4b^2) + (7a2+ab+-2b^2)$ 

$$\begin{array}{r} (2a^2 + 3ab + 4b^2) \\ +\ (7a^2 + ab + -2b^2) \\ \hline \boxed{\mathbf{9a^2 + 4ab + 2b^2}} \end{array}$$

Polynomials and Polynomial Functions

Multiplication

Multiplying Monomials by Monomials

Examples:

$$10x \cdot 9x = 90x^2$$

$$8x^3\left(-11x^7\right) = -88x^{10}$$

$$\left(-5x^4\right)(-x) = 5x^5$$

Polynomials and Polynomial Functions

Multiplication

Multiplying Monomials by Polynomials

Examples:

$$4x\left(x^2+4x+3\right)=4x^3+16x^2+12x$$

$$8x\left(7x^4+1\right)=56x^5+8x$$

$$\left(-5x^3\right)\left(3x^2-x+2\right)=-15x^5+5x^4-10x^3$$

Polynomials and Polynomial Functions

Multiplication

Multiplying Two Polynomials

Examples:

$$(x+5)(x^2+10x-3) = x^3+10x^2-3x+5x^2+50x-15$$

$$x^3+15x^2+47x-15$$

$$(4x^2+x+5)(3x-4) =$$

$$12x^3-16x^2+3x^2-4x+15x-20 =$$

$$12x^3-13x^2+11x-20$$

Multiplying Polynomials

Special Products

Multiplying Two Binomials using **FOIL**

First terms ➡ **O**uter terms ➡ **I**nner terms ➡ **L**ast terms

$$(x+3)(x+4) = x^2+4x+3x+12 =$$

$$x^2+7x+12$$

$$(x+7)(x-4) = x^2-4x+7x-28 =$$

$$x^2+3x-28$$

Multiplying Polynomials

Special Products

Multiplying Two Binomials using **FOIL**

First terms ➡ **O**uter terms ➡ **I**nner terms ➡ **L**ast terms

$$(2y^2+3)(y-4) = 2y^3-8y^2+3y-12 =$$

$$2y^3-8y^2+3y-12$$

$$(y^2+4)^2 = (y^2+4)(y^2+4) =$$

$$y^4+4y^2+4y^2+16 = y^4+8y^2+16$$

Multiplying Polynomials

Special Products

Squaring Binomials

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(4x-5)^2 = 16x^2 + 2(-20x) + 25 =$$

$$16x^2 - 40x + 25$$

$$(-3x-6)^2 = 9x^2 + 2(18x) + 36 =$$

$$9x^2 + 36x + 36$$

Multiplying Polynomials

Special Products

Multiplying the Sum and Difference of Two Binomials

$$(a+b)(a-b)=a^2-b^2$$

$$(x+9)(x-9)=x^2-9x+9x-81=x^2-81$$

$$(3x+5)(3x-5)=9x^2-15x+15x-25=9x^2-25$$

$$(x+8)(x-8)=x^2-64$$

$$(5x+13)(5x-13)=25x^2-169$$

Multiplying Polynomials

Special Products

Dividing by a Monomial

$$\frac{a \pm b}{c} = \frac{a}{c} \pm \frac{b}{c} \quad \text{where } c \neq 0$$

$$\frac{21x^8 + 9x^6 - 12x^4}{3x^3} =$$

$$\frac{21x^8}{3x^3} + \frac{9x^6}{3x^3} - \frac{12x^4}{3x^3} =$$

$$7x^5 + 3x^3 - 4x$$

Polynomials and Polynomial Functions

Multiplication

Multiplying Two Polynomials

Examples:

$$(x+3)(2x^2-5x+4)=$$

$$2x^3-5x^2+4x+6x^2-15x+12=$$

$$2x^3+x^2-11x+12$$

Polynomials and Polynomial Functions

Multiplication

Multiplying Two Polynomials

Examples:

$$(x-3y+2)^2 =$$

$$(x-3y+2)(x-3y+2) =$$

$$x^2-3xy+2x-3xy+9y^2-6y+2x-6y+4 =$$

$$x^2-6xy+4x+9y^2-12y+4$$

Algebraic long division

Divide $2x^3 + 3x^2 - x + 1$ by $x + 2$

$x + 2$ is the divisor

$x + 2 \overline{)2x^3 + 3x^2 - x + 1}$

The quotient will be here.

$2x^3 + 3x^2 - x + 1$ is the dividend

MIDDLE TERM SPLITTING

Replace the middle term with our working numbers.

$$x^2 - 2x - 63$$

$$x^2 - 9x + 7x - 63$$

Group the terms.

$$(x^2 - 9x)\ (+ 7x - 63)$$

Factor out the GCF

$$x(x - 9) + 7(x - 9)$$

The parentheses are the same! Weeedoggie!

$$(x + 7)(x - 9)$$

THANK YOU