Reading Material

Subject : Mathematics

Class : X

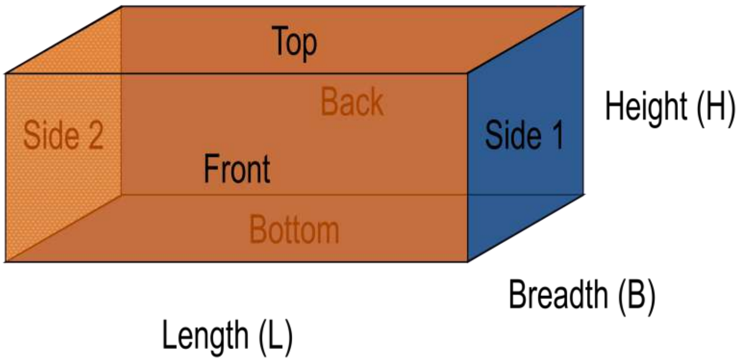
Chapter : Volumes

1.Volume:

- **Volume is the amount of space occupied by any 3-dimensional object**
- **Volume is measured in cubic units**
- **Volume of uniform solid = Base area X Height**

2.Volume of Cuboid:

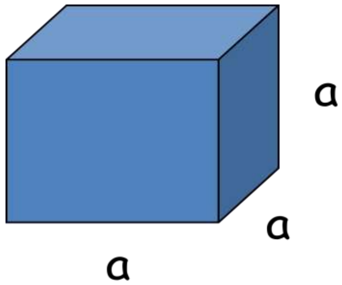
If l = length ,b= breadth and h= height



Volume of Cuboid = Base area X Height = lbh cubic units

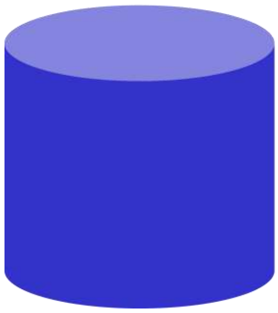
3.Volume of Cube :

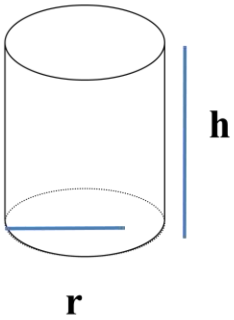
If side of the cube = a units



Volume of Cube = Base area X Height = a^3 cubic units

4.Volume of cylinder:

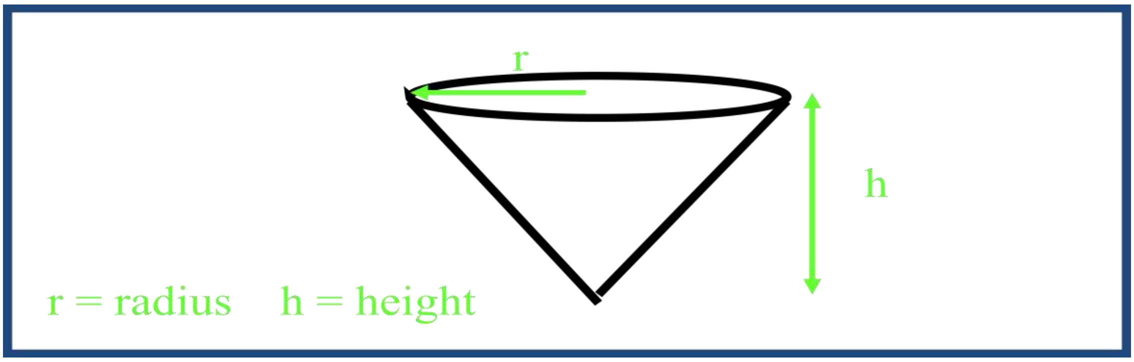




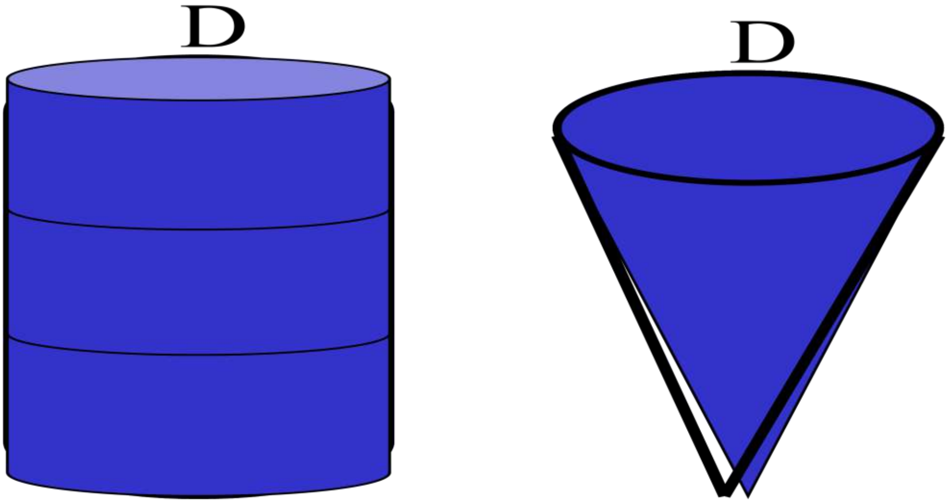
Volume of cylinder=Base area X Height

=∏ r^2h cubic units

5.Volume of Cone:



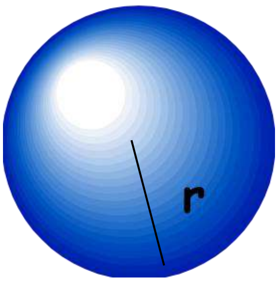
Volume of cone = $\frac{1}{3}$ of volume of cylinde (same r and h)



Volume of cone = $\frac{1}{3} \prod r^2 h$ cubic units

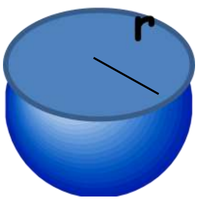
In cone relatiom between l ,h and r is $l^2 = h^2 + r^2$

6.Volume of Sphere:



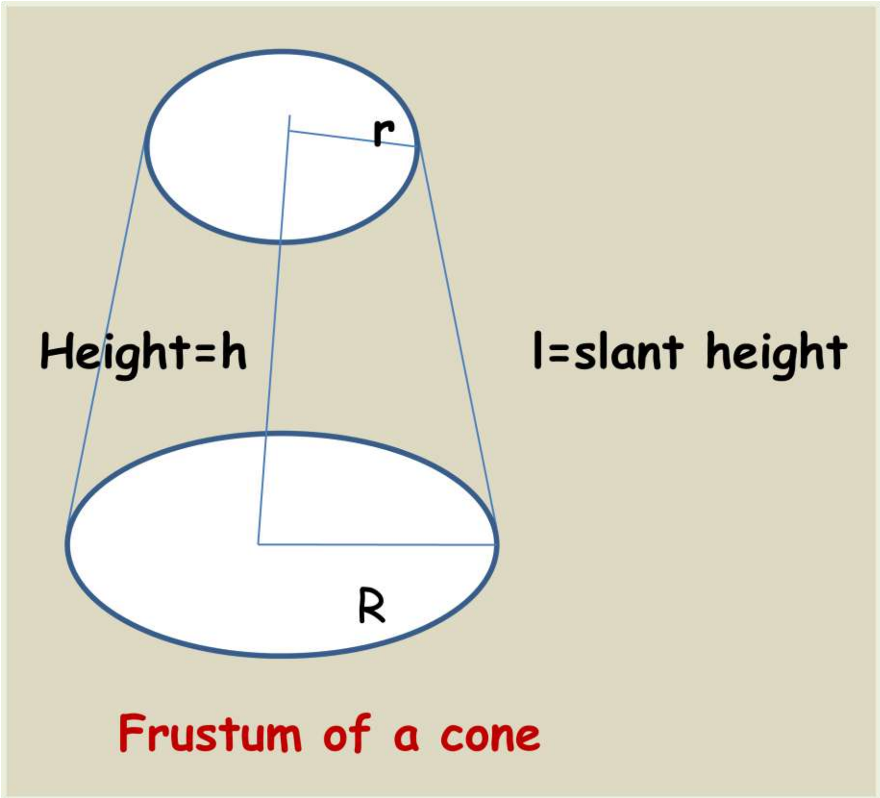
Volume of Sphere = $\frac{4}{3} \prod r^3$ cubic units

7.Volume of Hemi sphere:



Volume of of Hemi Sphere = $\frac{2}{3}\prod r^3$ cubic units

8.Volume of frustum of a cone:



Volume of frustum of a cone = $\frac{1}{3}\prod (r^2+R^2+rR)h$ cubic units

Relation between l,h,r,R is $l^2= h^2+ (R-r)^2$

9.cobination of 3d shapes :

Volume of combination of different solids isobtained by the sum of volumes of individual solids present

10.Conversion of Volumes:

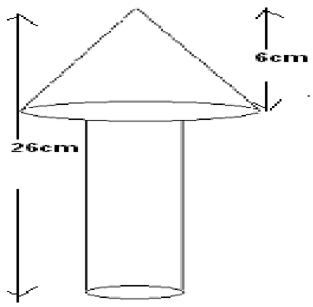
If one shape is converted into another shape then volume remais same

11.Example Problems for practice

13. SURFACE AREAS AND VOLUMES

TEXT BOOK

1.A wooden toy rocket is in the shape of a cone mounted on a cylinder. The height of the entire rocket is 26cm, while the height of the conical part is 6cm.the base of the conical portion has a diameter of 5cm, while the base diameter of cylindrical portion is 3cm. If the conical portion is to be painted orange and cylindrical portion yellow. Find the area of the rocket painted with each of these colours.



SOL:Denote radius of cone = r_1

Radius of cylinder = r_2

Slant height of cone $\ell = \sqrt{h^2 + r^2}$ ($r_1 = \frac{5}{2} = 2.5cm$;h=6cm)

$= \sqrt{2.5^2 + 6^2}$

$= \sqrt{6.25 + 36}$

$= \sqrt{42.25}$

$= 6.5cm$

the area to be painted orange = curved surface area of the cone + base area of the cone – base area of the cylinder

$= \Pi rl + \Pi r^2_1 - \Pi r^2_2$

$= \Pi [(2.5 \times 6.5) + (2.5)^2 - (1.5^2)]$

$= \Pi [20.25]$

$= 3.14 \times 20.25$

$= 63.585 \text{ cm}^2$

The area to be painted yellow =curved surface area of cylinder + Area of one base of the cylinder

$= 2 \Pi r_2 h_2 + \Pi r_2^2$

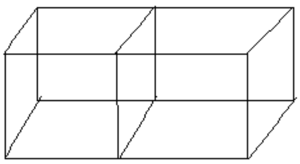
$= \Pi r_2 (2h_2 + r_2)$

$= (3.14 \times 1.5)(2 \times 20 + 1.5)$

$= 4.71 \times 41.5$

$= 195.465 \text{ cm}^2$

2. 2 cubes each of volume 64 cm³are joined end to end. Find the surface area of the resulting cuboid.



SOL:Volume of cube = 64 cm³

$a^3 = 64$

a = $\sqrt[3]{64}$

a = $\sqrt[3]{4X4X4}$

a = 4 cm

surface area of cuboid = 2 (lb + bh + hl) (l=8 cm , b=4 cm , h=4 cm)

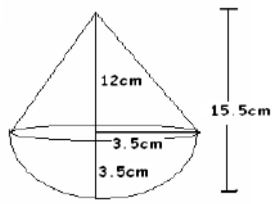
= 2 [(8 x 4) + (4 x 4) + (4 x 8)]

= 2 [32 + 16 + 32]

= 2 [80]

= 160 cm²

3. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.



SOL: r =3.5= $\frac{7}{2} cm$; h=12cm

Slant height of the cone $\ell = \sqrt{h^2 + r^2}$

$= \sqrt{\left(\frac{7}{2}\right)^2 + 12^2}$

$= \sqrt{\frac{49}{4} + 144}$

$= \sqrt{\frac{625}{4}}$

$= \frac{25}{2}$

$= 12.5cm$

The total surface area of toy = curved surface area of cone + curved surface area of hemisphere

$= \Pi r \ell + 2\Pi r^2$

$= \Pi r (\ell + 2r)$

$= \frac{22}{7} \times \frac{7}{2}\left[12.5 + \left(2 \times \frac{7}{2}\right)\right]$

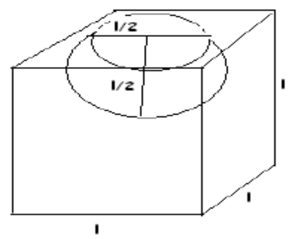
= 11 x 19.5

= 214.5 cm^2

4. A hemisphere depression is cut out form one face of a cubical wooden block such that the diameter ℓ of the the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

SOL: Required area= Surface area of 5faces of a cube + curved surface area of hemisphere + area of shaded region.

$= 5\ell^2 + 2\pi\left(\frac{\ell}{2}\right)^2 + \left[\ell^2 - \pi\left(\frac{l}{2}\right)^2\right]$



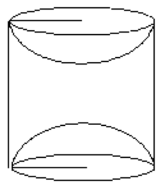
$= 6\ell^2 + \pi\left(\frac{\ell}{2}\right)^2$

$= 6\ell^2 + \frac{\pi\ell^2}{4}$

$= \frac{24\ell^2 + \pi\ell^2}{4}$

$= \frac{1}{4}\ell^2(24 + \pi)$

5. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm , find the total surface area of the article.



SOL: h=10cm; r =3.5cm= $\frac{7}{2}cm$

Total surface area of solid = curved surface area of cylinder + cuved surface area of 2 hemispheres

$= 2\Pi rh + 2(2\Pi r^2)$

$= 2\Pi r (h+2r)$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times [10 + (2 \times \frac{7}{2})]$

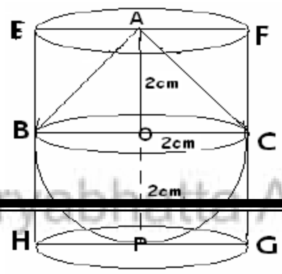
$= 22 \times 17$

= 374 sq.cm

6.A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2cm and the diameter of the base is 4cm. Determine the volume of the toy , find the difference of the volumes of the cylinder the toy.

SOL: Volume of toy =Volume of hemisphere + Volume of cone

Volume of the toy = $\frac{2}{3}\Pi r^3 + \frac{1}{3}\Pi r^2 h \quad [\because r = 2cm; h = 2cm]$



$= [\frac{2}{3} \times 3.14 \times (2)^3] + [\frac{1}{3} \times 3.14 \times (2)^2 \times 2]$

$= [\frac{2}{3} \times 3.14 \times 8] + [\frac{1}{3} \times 3.14 \times 8]$

$= (3.14 \times 8)[\frac{2}{3} + \frac{1}{3}]$

$= 25.12cm^3$

volume required = Volume of right circular cylinder – volume of the toy

$= \pi r^2 h - 25.12$ ($\because$ Height of Cylinder =2+2= 4cm ; r=2cm)

$= [3.14 \times (2)^2 \times 4] - 25.12$

$= 50.24 - 25.12$

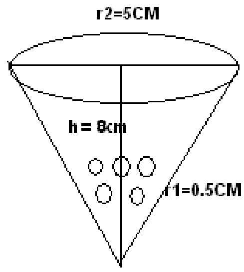
$= 25.12 cm^3$

$\therefore$ The required difference of two volumes = 25.12 cm^3

7. A vessel is in the form of an inverted cone. Its height is 8cm and the radius of its top which is open is 5cm. It is filled with water up to the brim. When lead shoats each of which is a sphere of radius 0.5cm are dropped in to the vessel, one fourth of the water flows out. Find the number of lead shots dropped in the vessel.

SOL: Let the number of led shots = n

n (volume of the lead shots) = ¼ volume of the cone



$n(\frac{4}{3}\Pi r_1^3) = \frac{1}{4}[\frac{1}{3}\Pi r_2^2 h]$

$(\because r_1 = 0.5cm; r_2 = 5cm; h = 10cm)$

$n \times \frac{4}{3} \times 0.5 \times 0.5 \times 0.5 = \frac{1}{4} \times \frac{1}{3} \times 5 \times 5 \times 8$

$n = \frac{(1/4)\times(1/3)\times 5\times 5\times 8}{(4/3)\times 0.5\times 0.5\times 0.5}$

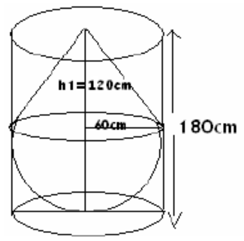
n = 100

No of lead shots dropped in the cone = 100

8. A solid consisting of a right circular cone of height 120cm and radius 60cm is standing on hemisphere of radius 60cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder if the radius of cylinder is 60cm and its height is 180cm.

SOL: $r = 60cm; h_1 = 120cm; h_2 = 180cm.$

Volume of water left in the cylinder = volume of the cylinder – vol of the cone – vol of the hemisphere



$= \Pi r^2 h_2 - \frac{1}{3}\Pi r^2 h_1 - \frac{2}{3}\Pi r^3$

$= \Pi r^2[h_2 - \frac{1}{3}h_1 - \frac{2}{3}r]$

$= \frac{22}{7} \times 60^2 [180 - (\frac{1}{3} \times 120) - (\frac{2}{3} \times 60)]$

$= \frac{22}{7} \times 3600[180 - 40 - 40]$

$= \frac{22}{7} \times 3600 \times 100$

$= 3.14 \times 3600 \times 100$

$= 314 \times 3600$

$= 1130400cm^3$

$= 1130400/100 \times 100 \times 100\ m^3$

$= 1.1304\ m^3$

9. A metallic sphere of radius 4.2 cm is melted and recast in to the shape of a cylinder of radius 6 cm. find the height of cylinder .

SOL:Volume of cylinder = volume of sphere

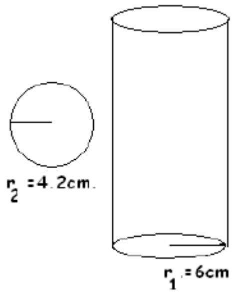
$$\Pi r_1^2 h = \frac{4}{3} \Pi r_2^3$$

$$6 \times 6 \times h = \frac{4}{3} \times 4.2 \times 4.2 \times 4.2$$

$$h = \frac{4}{3} \times 4.2 \times 4.2 \times 4.2 \times \frac{1}{6} \times \frac{1}{6}$$

$h = 5.6 \times 4.9$

$h = 27.44$cm

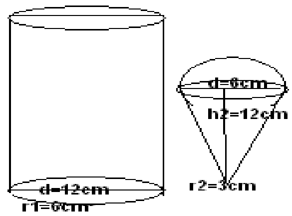


10. A container shaped like a right circular cylinder having diameter 12 cm and height 15cm is full of ice- cream. The ice-cream is to be filled in to cones of height 12cm and diameter 6cm, having a hemispherical shape on the top .Find the number of such cones which can be Filled with ice- cream.

Sol: $r_1 = 6cm; r_2 = 3cm; h_1 = 15cm; h_2 = 12cm.$

Let the number of cones required = n

n(volume of cone + volume of hemisphere)= volume of cylinder



$$n\left[\frac{1}{3} \Pi r_2^2 h_2 + \frac{2}{3} \Pi r_2^3\right] = \Pi r_1^2 h_1$$

$$n \times \frac{1}{3} \Pi r_2^2 [h_2 + 2 r_2] = \Pi r_1^2 h_1$$

$$n \times \frac{1}{3} \times 3 \times 3[12 + (2 \times 3)] = 6 \times 6 \times 15$$

$$n = 6 \times 6 \times 15 \times \frac{1}{3} \times \frac{1}{18}$$

$n = 10$

number of cones required = 10

11. A farmer connects a pipe of internal diameter 20cm from a canal in to a cylindrical tank in his field which is 10m in diameter and 2m deep. If water flows through the pipe at the rate of 3km/h in how much time will the tank be filled.

SOL: Diameter of a pipe = 20cm.

$$r = \frac{20}{2} = 10cm. \Rightarrow \frac{1}{10} m.$$

Speed of water = 3km/h

$$= 3000\text{m/h} = \frac{3000}{60} = 50 m/\min$$

Water tank has 2m.depth and diameter is 10m. i.e radius =5cm.

let time taken to fill the cylindrical tank = n minutes

volume of water that flows in n minutes= volume of cylinder

$$n \times \pi \times \left(\frac{1}{10}\right)^2 \times 50 = \pi \times 5^2 \times 2$$

$$n \times \frac{1}{100} \times 50 = 50$$

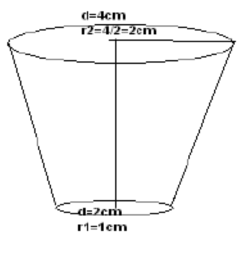
$$n = 100 \min.$$

Time taken ot fill cylindrical tank = 100min

12. A drinking glass is in the shape of a frustum of a cone height 14cm. The diameter of its two circular ends are 4cm and 2cm. Find the capacity of the glass.

SOL: Volume of glass= $\frac{1}{3} \Pi h (r_1^2 + r_1 r_2 + r_2^2)$

$$= \frac{1}{3} \times \frac{22}{7} \times 14(1^2 + (1 \times 2) + 2^2)$$



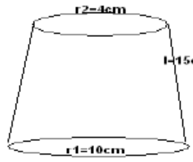
$= \frac{44}{3} \times (1 + 2 + 4)$

$= \frac{44}{3} \times 7 = \frac{308}{3}$

$= 102.6\ cm^3$

13. A fez the cap used by the truks is shaped like the frustum of a cone. If its radius on the open side is 10cm, radius at the upper base is 4cm and its slant height is 15cm , find the area of material used for making it.

SOL: $r_1 = 10cm.; r_2 = 4cm.; \ell = 15cm.$



Total surface area of fez= $\pi \ell (r_1 + r_2) + \pi r_2^2$

$= \frac{22}{7} \times 15(10 + 4) + \frac{22}{7} \times 4 \times 4$

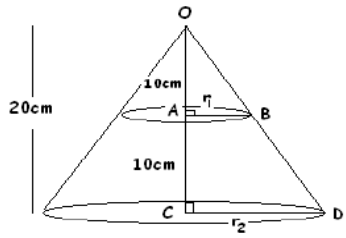
$= (44 \times 15) + (3.14 \times 16)$

$= 660 + 50.24$

$= 710.24\ cm^2$

14. A metallic right circular cone is 20cm high and whose vertical angle 60^0 is cut in to two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn in to a wire of diameter $\frac{1}{16} cm.$ find the length of wire.

SOL: In triangle OAB , $\angle A = 90^0$



$\frac{r_1}{10}$ = Tan 30^0

$\frac{r_1}{10} = \frac{1}{\sqrt{3}}$

$r_1 = \frac{10}{\sqrt{3}}$ cm.

In triangle OCD

$\frac{r_2}{20}$ = Tan 30^0

$\frac{r_2}{20} = \frac{1}{\sqrt{3}}$

$r_2 = \frac{20}{\sqrt{3}}$ cm.

Diameter of wire = $\frac{1}{16} cm.$

Radius of wire is $(r_3) = \frac{1}{16} \times \frac{1}{2} = \frac{1}{32} cm.$

$h_2 = 10$ cm.

. Volume of wire = Volume of frustum

$\Pi r_3^2 h_1 = \frac{1}{3} \Pi h_2 (r_1^2 + r_1 r_2 + r_2^2)$

$\frac{1}{32} \times \frac{1}{32} \times h_1 = \frac{1}{3} \times 10[(\frac{10}{\sqrt{3}})^2 + \frac{10}{\sqrt{3}} \times \frac{20}{\sqrt{3}} + (\frac{20}{\sqrt{3}})^2]$

$h_1 = \frac{1}{3} \times 10[(100+200+400) \times \frac{1}{3}] \times 32 \times 32$

$$= \frac{1}{3} \times 10 \times \frac{700}{3} \times 32 \times 32$$

$$= \frac{79644.4}{100} \text{ m.}$$

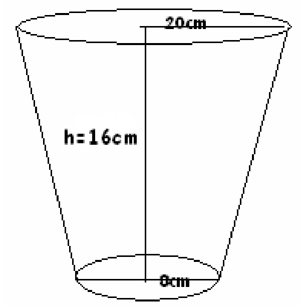
h_1 =7964.4 m

Length of the wire = 7964.4 m.

15. A well of diameter 3m is dug 14m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4m to form an embankment. Find the height of the embankment.(Try yourself)(Ans: 1.125m)

16. A container , opened from the top is made up a metal sheet is in the form of a frustum of a cone of height 16cm with radii of its lower and upper ends as 8cm and 20cm , respectively. Find the cost of the milk which can completely fill the container at the rate of Rs.20. per lit. Also find the cost of metal sheet used to make the container if its costs Rs.8 per 100 cm² .

SOL: $r_1 = 20cm.; r_2 = 8cm.; h = 16cm.$



volume of frustum= $\frac{1}{3}\Pi h\ (r_1^2 + r_1 r_2 + r_2^2)$

$= \frac{1}{3} \times \frac{22}{7} \times 16(20^2 + (20 \times 8) + 8^2)$

$\frac{1}{3} \times 3.14 \times 16(400+160+ 64)$

$=10449.92cm^3$

=10449.92/1000lits

= 10.449lit

cost of 1 lit milk = Rs. 20

Total cost of milk = 10.449 × 20

= Rs 208.98

Slant height of frustum =

$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$l = \sqrt{16^2 + (20-8)^2}$$

$$l = \sqrt{256 + 144}$$

$$l = \sqrt{400}$$

$$l = 20cm.$$

total surface area of frustum = $\pi\ell\ (r_1+r_2) + \pi\ r_2^2$

$= \frac{22}{7} \times 20 \times (20+8) + \frac{22}{7} \times 8^2$

$= \frac{22}{7} [20 \times 28 + 64]$

= 3.14 × 624

= 1959.36cm²

cost of 100 cm² = Rs 8

cost of 1959.36 cm² $= \frac{8 \times 1959.36}{100}$

=8 × 19.593

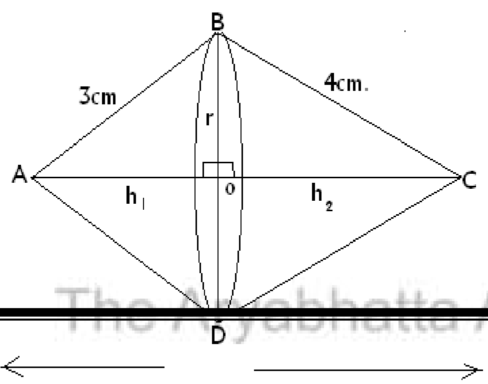
= Rs 156.74

The cost of the metal sheet used to make the container = Rs.156.74

17. A right triangle whose sides are 3cm and 4cm other than the hypotenuse is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed.

SOL:

in triangle ΔAOB & ΔABC



$\angle A = \angle A$ (common)
$\angle AOB = \angle ABC$ ($=90^0$)
$\Delta AOB \sim \Delta ABC$ (AA)

$\frac{r}{4} = \frac{3}{5}$

$r = \frac{12}{5}$ cm

In triangle ΔABC; $\angle B = 90^0$
$AC^2 = AB^2 + BC^2$
$AC^2 = 3^2 + 4^2 = 9+16 = 25$
$AC = \sqrt{25} = 5$cm.
Volume of double cone =Volume of cone A + volume of cone B

$= \frac{1}{3}\pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2$

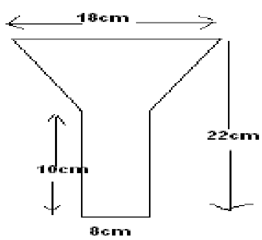
$= \frac{1}{3}\Pi r^2(h_1 + h_2)$ $\{\because h_1 + h_2 = 5cm\}$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{12}{5} \times \frac{12}{5} \times 5$
$= 3.14 \times 9.6$
$= 30.144 cm^3$

Total surface area of double cone = curved surface area of cone A + curved surface area of coneB
$= \pi r \ell_1 + \pi r \ell_2$ $(\because \ell_1 = 3cm;\ \ell_2 = 4cm)$
$= \Pi r(\ell_1 + \ell_2)$
$= \frac{22}{7} \times \frac{12}{5}(3+4)$
$= \frac{22}{7} \times \frac{12}{5} \times 7$
$= 22 \times 2.4$
$= 52.8\ cm^2$

18. An oil funnel made of tin sheet consists of a 10cm long cylindrical portion attached to a frustum of a cone if the total height 22cm diameter of the cylindrical portion is 8cm and the diameter of the top of the funnel is 18cm find the area of the tin sheet required to make the funnel.

SOL:Slant height



$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$
$$l = \sqrt{12^2 + (9-4)^2}$$
$$l = \sqrt{144 + 25}$$
$$l = \sqrt{169}$$
$$l = 13cm.$$
$$r_1 = 9cm.; r_2 = 4cm.; h = 10cm.$$

total surface area of funnel = curved surface area of frustum+curved surface area of cylinder

$= \Pi \ell (r_1 + r_2) + 2\Pi r h$

$= \frac{22}{7} \times 13\ (9+4) + 2 \times \frac{22}{7} \times 4 \times 10$

$= \frac{22}{7}[13 \times 13 + 2 \times 40]$

$= \frac{22}{7}[169+80]$

$= 3.14 \times 249$

$= 781.86cm^2$

19.A copper wire 4mmin diameter is evenly wound about a cylinder whose length is 24cm and diameter 20cm so as to cover the whole surface.Find the length and weight of the wire assuming the specific gravity to be 8.88 gm/cm³ (Try yourself).

OUT OF TEXT BOOK

1. A conical vessel of radius 6cm and height 8cm is completely filled with water. A sphere is lowered in to the water and its size is such that when it touches the sides, it just immersed. What fraction of water over flows?

SOL: In ΔO^1AV; $\angle O=90^0$

$AV^2=O^1A^2+O^1V^2$

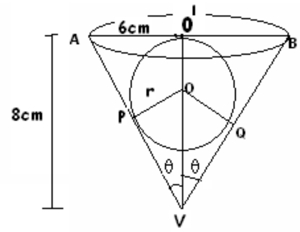
$AV^2=6^2+8^2$

$AV^2=100$

$AV=10cm$

Let the radius of the sphere= rcm

In ΔO^1AV



$\text{Tan } \theta = \frac{6}{8} = \frac{3}{4}$

$\text{Sin}\theta = \frac{6}{10} = \frac{3}{5}$

in ▲VPO we have

$\sin\theta = \frac{r}{VO}$

$\frac{3}{5} = \frac{r}{8-r}$

24-3r=5r

8r=24

$r = \frac{24}{3} = 3$ cm

volume of water over flows (v_1) =volume of the sphere

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 3^3$

$=36\pi cm^3$

volume of water(v_2) = volum e of the cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 6^2 \times 8$

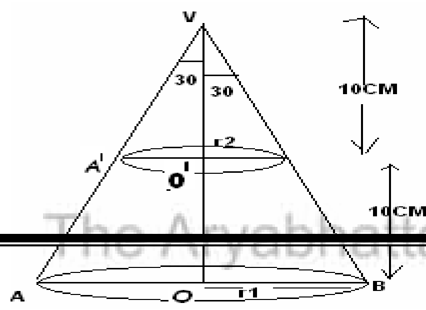
$=96\pi cm^3$

Fraction of water that flows out = $v_1 : v_2$

$= 36\pi : 96\pi$

$= 3 : 8$

2. A solid metallic right circular cone 20cm height with vertical angle 60is cut in to two parts the middle point of its height by a plane parallel to the base. If the frustum so obtained be drawn in to a wire of diameter 1/3mm ,find the length of the wire.



SOL: In triangle VOA and VO^1A^1 we have

Tan 30^0 = $\frac{OA}{VO}$

$$\frac{1}{\sqrt{3}} = \frac{r_1}{20}$$

$$r_1 = \frac{20}{\sqrt{3}}$$

Tan30^0=$\frac{O^1A^1}{VO^1}$

$$\frac{1}{\sqrt{3}} = \frac{r_2}{10}$$

$$r_2 = \frac{10}{\sqrt{3}}$$

Volume of frustum= $\frac{1}{3}$ Πh ($r_1^2 + r_1r_2 + r_2^2$)

$$= \frac{1}{3}\times\pi\times10\left[\left(\frac{20}{\sqrt{3}}\right)^2+\left(\frac{20}{\sqrt{3}}\times\frac{10}{\sqrt{3}}\right)+\left(\frac{10}{\sqrt{3}}\right)^2\right]$$

$$= \frac{10\pi}{3}\times\left[\frac{400}{3}+\frac{200}{3}+\frac{100}{3}\right]$$

$$= \frac{7000\pi}{9}cm^2$$

let the length of the wire of $\frac{1}{3}$ mm diameter be 1cm. then

$$d = \frac{1}{3}mm; r = \frac{1}{3}\times\frac{1}{2} = \frac{1}{6}mm \Rightarrow \frac{1}{60}cm$$

Volume of metal used in wire = $\pi r^2 h \quad (\because h = \ell)$

$$= \pi\times\left(\frac{1}{60}\right)^2\times\ell$$

$$= \frac{\pi\ell}{3600}$$

Volume of metal used in wire = volume of the frustum

$$\frac{\pi\ell}{3600} = \frac{7000\pi}{9}$$

$$\ell = \frac{7000\pi}{9}\times\frac{3600}{\pi}$$

ℓ = (7000 × 400)cm

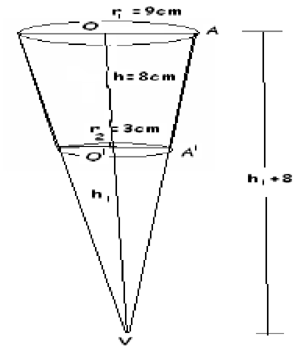
ℓ = $\left(\frac{7000\times400}{100}\right)m$

ℓ = 28,000m

3. A bucket of height 8cm and made up of copper sheet is in the form of a frustum of a right circular cone with radii of its lower and upper ends are 3cm &9cm respectively. calculate (i)the height of the cone of which the bucket is a part (ii)the volume of the water which can be filled in the bucket..

SOL: h=8cm r_1=9cm r_2 =3cm

(i)Let (h_1+8) the height of the cone of which the bucket is a part.



In figure: Δ VOA ~ Δ VO'A'

$$\frac{h_1+8}{h_1}=\frac{r_1}{r_2}$$

$$1+\frac{8}{h_1}=\frac{9}{3}$$

$$1+\frac{8}{h_1}=3$$

$$\frac{8}{h_1}=2$$

$$h_1=4$$

the height of the cone =(h_1 + 8) =4+ 8=12cm

(ii) Vol of the water which can be filled in the bucket =volume of the frustum

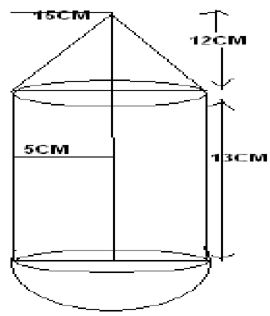
$$=\frac{1}{3}\pi h\,(r_1^2 + r_1r_2 + r_2^2)$$

$$=\frac{1}{3}\times\frac{22}{7}\times 8\left(9^2+9\times 3+3^2\right)$$

$$=312\pi \text{ cm}^3$$

4. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The height and radius of the cylindrical part are 13cm & 5cm respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part calculate the surface area of the toy, if height of the conical part is 12cm.

SOL:



r=5cm h=12cm

Slant height

$$\ell=\sqrt{h^2+r^2}$$
$$\ell=\sqrt{5^2+12^2}$$
$$\ell=\sqrt{169}$$
$$\ell=13cm$$

Surface area of toy = c s a of cylindrical part + c s a of hemispherical part +c s a of conical part

$$=\pi r\ell+2\pi rh+2\pi r^2$$
$$=\pi r(\ell+2h+2r)$$
$$=\frac{22}{7}\times 5\times\left(13+2\times 13+2\times 5\right)$$

$= \frac{22}{7} \times 5(13+26+10)$

$=22 \times 5 \times 7$

$=770cm^2$

5. A hallow cone is cut by a plane parallel to the base and upper portion is removed. If the curved surface area of the reminder is 8/9 of the curved surface area of whole cone, find the ratio of the line segment into which the cones altitude is divided by the plane.

SOL: In figure: Δ VOA ~ Δ VO^1A^1

$$\frac{h_1}{h_1+h_2} = \frac{r_1}{r_2} = \frac{\ell_1}{\ell_2}$$

Let the required ratio = $h_1 : h_2$

Curved surface area of the frustum= $\frac{8}{9} \times$ (c s a of the big cone)

Curved surface area of small cone + c s a of frustum = c s a of big cone

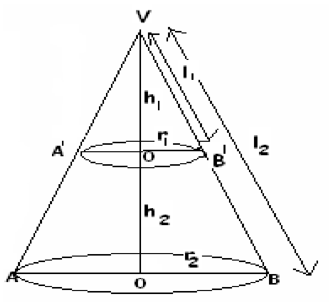
Curved surface area of small cone + $\frac{8}{9} \times$ (c s a of big cone) = c s a of big cone

Curved surface area of small cone= c s a of big cone – $\frac{8}{9} \times$ (c s a of big cone)

Curved surface area of small cone = c s a of big cone (1- $\frac{8}{9}$)

Curved surface area of small cone = $\frac{1}{9} \times$ (c s a of big cone)

$\pi r_1 \ell_1 = \frac{1}{9} (\pi r_2 \ell_2)$



$\frac{r_1}{r_2} \times \frac{\ell_1}{\ell_2} = \frac{1}{9}$

$\left(\frac{h_1}{h_1+h_2}\right) \times \left(\frac{h_1}{h_1+h_2}\right) = \frac{1}{9}$

$\left(\frac{h_1}{h_1+h_2}\right)^2 = \frac{1}{9}$

$\frac{h_1}{h_1+h_2} = \frac{1}{3}$

$3h_1 = h_1 + h_2$

$3h_1 - h_1 = h_2$

$2h_1 = h_2$

$\frac{h_1}{h_2} = \frac{1}{2}$

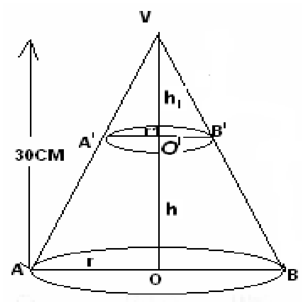
$\therefore h_1 : h_2 = 1 : 2$

6. The height of the cone is 30cm a small cone is cut of at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made.

SOL: In figure: Δ VOA ~ Δ VO^1A^1

$$\therefore \frac{VO}{VO^1} = \frac{OA}{O^1A^1}$$

$$\frac{30}{h_1} = \frac{r}{r_1}$$



Let the section made at the height = h cm

Volume of small cone = $\frac{1}{27}$ × (volume of big cone)

$$\frac{1}{3}\pi r_1^{\,2} h_1 = \frac{1}{27} \times \frac{1}{3}\pi r^2 \times 30$$

$$\left(\frac{r_1}{r}\right)^2 \times h_1 = \frac{10}{9}$$

$$\left(\frac{h_1}{30}\right)^2 \times h_1 = \frac{10}{9}$$

$$h_1^{\,3} = 1000$$

$$h_1 = 10cm$$

$$\therefore h = 30 - h_1 = 30 - 10 = 20cm$$

Hence, 20cm high above the base is the section made.