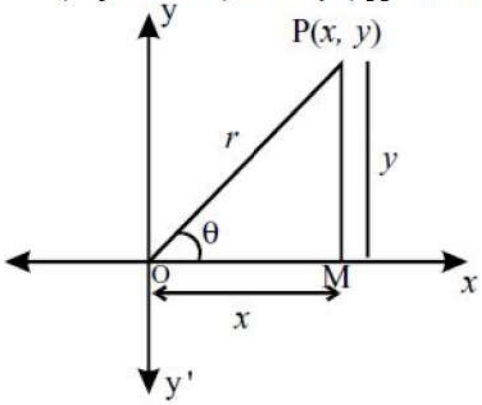
CLASS X: CHAPTER - 8
INTRODUCTION TO TRIGONOMETRY

IMPORTANT FORMULAS & CONCEPTS

The word 'trigonometry' is derived from the Greek words 'tri' (meaning three), 'gon' (meaning sides) and 'metron' (meaning measure). In fact, **trigonometry** is the study of relationships between the sides and angles of a triangle.

Trigonometric Ratios (T - Ratios) of an acute angle of a right triangle

In XOY-plane, let a revolving line OP starting from OX, trace out ∠XOP=θ.
From P (x, y) draw PM ⊥ to OX.
In right angled triangle OMP. OM = x (Adjacent side); PM = y (opposite side); OP = r (hypotenuse).



$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{y}{r} \qquad \cos ec\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{r}{y}$$

$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{x}{r} \qquad \sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{r}{x}$$

$$\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent Side}} = \frac{y}{x} \qquad \cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite side}} = \frac{x}{y}$$

Reciprocal Relations

$$\sin\theta = \frac{1}{\cos ec\theta} \qquad \cos ec\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Quotient Relations

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \textit{and} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

- **Remark 1 :** sin q is read as the "sine of angle q" and it should never be interpreted as the product of 'sin' and 'q'
- **Remark 2 : Notation :** $(\sin\theta)^2$ is written as $\sin^2\theta$ (read "sin square q'') Similarly $(\sin\theta)^n$ is written as $\sin^n\theta$ (read ''sin nth power q''), n being a positive integer.
- **Note :** $(\sin\theta)^2$ should not be written as $\sin\theta^2$ or as $\sin^2\theta^2$
- **Remark 3 :** Trigonometric ratios depend only on the value of θ and are independent of the lengths of the sides of the right angled triangle.

Trigonometric ratios of Complementary angles.

$\sin(90-\theta) = \cos\theta$ $\quad\quad$ $\cos(90-\theta) = \sin\theta$

$\tan(90-\theta) = \cot\theta$ $\quad\quad$ $\cot(90-\theta) = \tan\theta$

$\sec(90-\theta) = \operatorname{cosec}\theta$ $\quad\quad$ $\operatorname{cosec}(90-\theta) = \sec\theta$.

Trigonometric ratios for angle of measure.

0^0, 30^0,45^0, 60^0 and 90^0 in tabular form.

$\angle A$	0^0	30^0	45^0	60^0	90^0
sinA	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosA	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanA	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosecA	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
secA	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cotA	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratios of an angle is said to be a trigonometric identity if it is satisfied for all values of θ for which the given trigonometric ratios are defined.

Identity (1) : $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$.

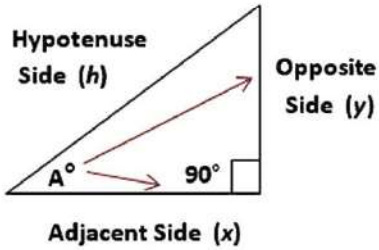
Identity (2) : $\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$ and $\tan^2\theta = \sec^2\theta - 1$.

Identity (3) : $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta = 1$ and $\cot^2\theta = \operatorname{cosec}^2\theta - 1$.

SOME TIPS

Right Triangle	SOH-CAH-TOA Method	Coordinate System Method
	**SOH:** $\text{sine}(A) = \textbf{sin}(A) = \frac{\text{Opposite}}{\text{Hypotenuse}}$ **CAH:** $\text{cosine}(A) = \textbf{cos}(A) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$ **TOA:** $\text{tangent}(A) = \textbf{tan}(A) = \frac{\text{Opposite}}{\text{Adjacent}}$ $\text{cosecant}(A) = \textbf{csc}(A) = \frac{1}{\sin(A)} = \frac{\text{Hypotenuse}}{\text{Opposite}}$ $\text{secant}(A) = \textbf{sec}(A) = \frac{1}{\cos(A)} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$ $\text{cotangent}(A) = \textbf{cot}(A) = \frac{1}{\tan(A)} = \frac{\text{Adjacent}}{\text{Opposite}}$	$\textbf{sin}(A) = \frac{y}{h}$ $\textbf{cos}(A) = \frac{x}{h}$ $\textbf{tan}(A) = \frac{y}{x}$ $\textbf{csc}(A) = \frac{1}{\sin(A)} = \frac{h}{y}$ $\textbf{sec}(A) = \frac{1}{\cos(A)} = \frac{h}{x}$ $\textbf{cot}(A) = \frac{1}{\tan(A)} = \frac{x}{y}$

Each trigonometric function in terms of the other five.

in terms of	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$\sin\theta =$	$\sin\theta$	$\pm\sqrt{1-\cos^2\theta}$	$\pm\frac{\tan\theta}{\sqrt{1+\tan^2\theta}}$	$\frac{1}{\csc\theta}$	$\pm\frac{\sqrt{\sec^2\theta-1}}{\sec\theta}$	$\pm\frac{1}{\sqrt{1+\cot^2\theta}}$
$\cos\theta =$	$\pm\sqrt{1-\sin^2\theta}$	$\cos\theta$	$\pm\frac{1}{\sqrt{1+\tan^2\theta}}$	$\pm\frac{\sqrt{\csc^2\theta-1}}{\csc\theta}$	$\frac{1}{\sec\theta}$	$\pm\frac{\cot\theta}{\sqrt{1+\cot^2\theta}}$
$\tan\theta =$	$\pm\frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$	$\pm\frac{\sqrt{1-\cos^2\theta}}{\cos\theta}$	$\tan\theta$	$\pm\frac{1}{\sqrt{\csc^2\theta-1}}$	$\pm\sqrt{\sec^2\theta-1}$	$\frac{1}{\cot\theta}$
$\csc\theta =$	$\frac{1}{\sin\theta}$	$\pm\frac{1}{\sqrt{1-\cos^2\theta}}$	$\pm\frac{\sqrt{1+\tan^2\theta}}{\tan\theta}$	$\csc\theta$	$\pm\frac{\sec\theta}{\sqrt{\sec^2\theta-1}}$	$\pm\sqrt{1+\cot^2\theta}$
$\sec\theta =$	$\pm\frac{1}{\sqrt{1-\sin^2\theta}}$	$\frac{1}{\cos\theta}$	$\pm\sqrt{1+\tan^2\theta}$	$\pm\frac{\csc\theta}{\sqrt{\csc^2\theta-1}}$	$\sec\theta$	$\pm\frac{\sqrt{1+\cot^2\theta}}{\cot\theta}$
$\cot\theta =$	$\pm\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$	$\pm\frac{\cos\theta}{\sqrt{1-\cos^2\theta}}$	$\frac{1}{\tan\theta}$	$\pm\sqrt{\csc^2\theta-1}$	$\pm\frac{1}{\sqrt{\sec^2\theta-1}}$	$\cot\theta$

Note: $\csc\theta$ **is same as** $\cos ec\theta$**.**

MCQ WORKSHEET-I
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1. In ΔOPQ, right-angled at P, OP = 7 cm and OQ – PQ = 1 cm, then the values of sin Q.
(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) 1 (d) none of the these

2. If $\sin A = \frac{24}{25}$, then the value of cosA is
(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) 1 (d) none of the these

3. In ΔABC, right-angled at B, AB = 5 cm and $\angle ACB = 30°$ then the length of the side BC is
(a) $5\sqrt{3}$ (b) $2\sqrt{3}$ (c) 10 cm (d) none of these

4. In ΔABC, right-angled at B, AB = 5 cm and $\angle ACB = 30°$ then the length of the side AC is
(a) $5\sqrt{3}$ (b) $2\sqrt{3}$ (c) 10 cm (d) none of these

5. The value of $\frac{2\tan 30^0}{1+\tan^2 30^0}$ is
(a) sin 60° (b) cos 60° (c) tan 60° (d) sin 30°

6. The value of $\frac{1-\tan^2 45^0}{1+\tan^2 45^0}$ is
(a) tan 90° (b) 1 (c) sin 45° (d) 0

7. sin 2A = 2 sin A is true when A =
(a) 0° (b) 30° (c) 45° (d) 60°

8. The value of $\frac{2\tan 30^0}{1-\tan^2 30^0}$ is
(a) sin 60° (b) cos 60° (c) tan 60° (d) sin 30°

9. $9\sec^2 A - 9\tan^2 A =$
(a) 1 (b) 9 (c) 8 (d) 0

10. $(1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A) =$
(a) 0 (b) 1 (c) 2 (d) –1

11. $(\sec A + \tan A)(1 - \sin A) =$
(a) sec A (b) sin A (c) cosec A (d) cos A

12. $\frac{1+\tan^2 A}{1+\cot^2 A} =$
(a) $\sec^2 A$ (b) –1 (c) $\cot^2 A$ (d) $\tan^2 A$

MCQ WORKSHEET-II
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1. If $\sin 3A = \cos (A - 26°)$, where 3A is an acute angle, find the value of A.
(a) 29^0 (b) 30^0 (c) 26^0 (d) 36^0

2. If $\tan 2A = \cot (A - 18°)$, where 2A is an acute angle, find the value of A.
(a) 29^0 (b) 30^0 (c) 26^0 (d) none of these

3. If $\sec 4A = \operatorname{cosec} (A - 20°)$, where 4A is an acute angle, find the value of A.
(a) 22^0 (b) 25^0 (c) 26^0 (d) none of these

4. The value of $\tan 48° \tan 23° \tan 42° \tan 67°$ is
(a) 1 (b) 9 (c) 8 (d) 0

5. If ΔABC is right angled at C, then the value of $\cos(A + B)$ is
(a) 0 (b) 1 (c) ½ (d) n.d.

6. The value of the expression $\left[\frac{\sin^2 22^0 + \sin^2 68^0}{\cos^2 22^0 + \cos^2 68^0} + \sin^2 63^0 + \cos 63^0 \sin 27^0\right]$ is
(a) 3 (b) 0 (c) 1 (d) 2

7. If $\cos A = \frac{24}{25}$, then the value of sinA is
(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) 1 (d) none of the these

8. If ΔABC is right angled at B, then the value of $\cos(A + C)$ is
(a) 0 (b) 1 (c) ½ (d) n.d.

9. If $\tan A = \frac{4}{3}$, then the value of cosA is
(a) $\frac{3}{5}$ (b) $\frac{4}{3}$ (c) 1 (d) none of the these

10. If ΔABC is right angled at C, in which AB = 29 units, BC = 21 units and $\angle ABC = \alpha$. Determine the values of $\cos^2\alpha + \sin^2\alpha$ is
(a) 0 (b) 1 (c) ½ (d) n.d.

11. In a right triangle ABC, right-angled at B, if $\tan A = 1$, then the value of $2 \sin A \cos A =$
(a) 0 (b) 1 (c) ½ (d) n.d.

12. Given $15 \cot A = 8$, then $\sin A =$
(a) $\frac{3}{5}$ (b) $\frac{4}{3}$ (c) 1 (d) none of the these

MCQ WORKSHEET-III
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1. In a triangle PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm, then the value of sin P is

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) 1 (d) none of the these

2. In a triangle PQR, right-angled at Q, PQ = 3 cm and PR = 6 cm, then $\angle QPR$ =

(a) 0° (b) 30° (c) 45° (d) 60°

3. If $\sin(A - B) = \frac{1}{2}$ and $\cos(A + B) = \frac{1}{2}$, then the value of A and B, respectively are

(a) 45° and 15° (b) 30° and 15° (c) 45° and 30° (d) none of these

4. If sin (A – B) = 1 and cos(A + B) = 1, then the value of A and B, respectively are

(a) 45° and 15° (b) 30° and 15° (c) 45° and 30° (d) none of these

5. If $\tan(A - B) = \frac{1}{\sqrt{3}}$ and $\tan(A + B) = \sqrt{3}$, then the value of A and B, respectively are

(a) 45° and 15° (b) 30° and 15° (c) 45° and 30° (d) none of these

6. If $\cos(A - B) = \frac{\sqrt{3}}{2}$ and sin (A + B) = 1, then the value of A and B, respectively are

(a) 45° and 15° (b) 30° and 15° (c) 60° and 30° (d) none of these

7. The value of $2\cos^2 60^0 + 3\sin^2 45^0 - 3\sin^2 30^0 + 2\cos^2 90^0$ is

(a) 1 (b) 5 (c) 5/4 (d) none of these

8. sin 2A = 2 sin AcosA is true when A =

(a) 0° (b) 30° (c) 45° (d) any angle

9. sin A = cosA is true when A =

(a) 0° (b) 30° (c) 45° (d) any angle

10. If $\sin A = \frac{1}{2}$, then the value of $3\cos A - 4\cos^3 A$ is

(a) 0 (b) 1 (c) ½ (d) n.d.

11. If 3cotA = 4, then the value of $\cos^2 A - \sin^2 A$ is

(a) $\frac{3}{4}$ (b) $\frac{7}{25}$ (c) ½ (d) $\frac{24}{25}$

12. If 3tanA = 4, then the value of $\frac{3\sin A + 2\cos A}{3\sin A - 2\cos A}$ is

(a) 1 (b) $\frac{7}{25}$ (c) 3 (d) $\frac{24}{25}$

MCQ WORKSHEET-IV
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1. Value of θ, for $\sin 2\theta = 1$, *where* $0^0 < \theta < 90^0$ is:
(a) 30° (b) 60° (c) 45° (d) 135^0.

2. Value of $\sec^2 26^0 - \cot^2 64^0$ is:
(a) 1 (b) –1 (c) 0 (d) 2

3. Product $\tan 1^0 . \tan 2^0 . \tan 3^0 \ldots\ldots \tan 89^0$ is:
(a) 1 (b) –1 (c) 0 (d) 90

4. $\sqrt{1+\tan^2\theta}$ is equal to:
(a) $\cot\theta$ (b) $\cos\theta$ (c) $\cos ec\theta$ (d) $\sec\theta$

5. If $A + B = 90^0$, $\cot B = \frac{3}{4}$ then tanA is equal to;
(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

6. Maximum value of $\frac{1}{\cos ec\theta}$, $0^0 < \theta < 90^0$ is:
(a) 1 (b) –1 (c) 2 (d) $\frac{1}{2}$

7. If $\cos\theta = \frac{1}{2}$, $\sin\phi = \frac{1}{2}$ then value of $\theta + \phi$ is
(a) 30° (b) 60° (c) 90° (d) 120^0.

8. If Sin (A + B) = 1 = cos(A – B) then
(a) $A = B = 90^0$ (b) $A = B = 0^0$ (c) $A = B = 45^0$ (d) A = 2B

9. The value of $\sin 60^0 \cos 30^0 - \cos 60^0 \sin 30^0$ is
(a) 1 (b) –1 (c) 0 (d) none of these

10. The value of $2\sin^2 30^0 - 3\cos^2 45^0 + \tan^2 60^0 + 3\sin^2 90^0$ is
(a) 1 (b) 5 (c) 0 (d) none of these

PRACTICE QUESTIONS

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TRIGONOMETRIC RATIOS

1. If $\tan\theta = \frac{1}{\sqrt{5}}$, what is the value of $\frac{\cos ec^2\theta - \sec^2\theta}{\cos ec^2\theta + \sec^2\theta}$?

2. If $\sin\theta = \frac{4}{5}$, find the value of $\frac{\sin\theta\tan\theta - 1}{2\tan^2\theta}$.

3. If $\cos A = \frac{1}{2}$, find the value of $\frac{2\sec A}{1+\tan^2 A}$.

4. If $\sin\theta = \frac{\sqrt{3}}{2}$, find the value of all T– ratios of θ.

5. If $\cos\theta = \frac{7}{25}$, find the value of all T– ratios of θ.

6. If $\tan\theta = \frac{15}{8}$, find the value of all T– ratios of θ.

7. If $\cot\theta = 2$, find the value of all T– ratios of θ.

8. If $\text{cosec}\,\theta = \sqrt{10}$, find the value of all T– ratios of θ.

9. If $\tan\theta = \frac{4}{3}$, show that $(\sin\theta + \cos\theta) = \frac{7}{5}$.

10. If $\sec\theta = \frac{5}{4}$, show that $\frac{(\sin\theta - 2\cos\theta)}{(\tan\theta - \cot\theta)} = \frac{12}{7}$.

11. If $\tan\theta = \frac{1}{\sqrt{7}}$, show that $\frac{(\cos ec^2\theta - \sec^2\theta)}{(\cos ec^2\theta + \sec^2\theta)} = \frac{3}{4}$.

12. If $\cos ec\theta = 2$, show that $\left\{\cot\theta + \frac{\sin\theta}{1+\cos\theta}\right\} = 2$.

13. If $\sec\theta = \frac{5}{4}$, verify that $\frac{\tan\theta}{(1+\tan^2\theta)} = \frac{\sin\theta}{\sec\theta}$.

14. If $\cos\theta = 0.6$, show that $(5\sin\theta - 3\tan\theta) = 0$.

15. In a triangle ACB, right-angled at C, in which AB = 29 units, BC = 21 units and $\angle ABC = \theta$. Determine the values of (i) $\cos^2\theta + \sin^2\theta$ (ii) $\cos^2\theta - \sin^2\theta$

16. In a triangle ABC, right-angled at B, in which AB = 12 cm and BC = 5cm. Find the value of cosA, cosecA, cosC and cosecC.

17. In a triangle ABC, $\angle B = 90^0$, AB = 24 cm and BC = 7 cm. Find (i) SinA, CosA (ii) SinC, CosC.

PRACTICE QUESTIONS
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T – RATIOS OF SOME PARTICULAR ANGLES

Evaluate each of the following:

1. $\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$

2. $\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$

3. $\cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$

4. $\sin 60^0 \sin 45^0 - \cos 60^0 \cos 45^0$

5. $\dfrac{\sin 30^0}{\cos 45^0} + \dfrac{\cot 45^0}{\sec 60^0} - \dfrac{\sin 60^0}{\tan 45^0} - \dfrac{\cos 30^0}{\sin 90^0}$

6. $\dfrac{\tan^2 60^0 + 4\cos^2 45^0 + 3\cos ec^2 60^0 + 2\cos^2 90^0}{2\cos ec 30^0 + 3\sec 60^0 - \dfrac{7}{3}\cot^2 30^0}$

7. $4(\sin^4 30^0 + \cos^4 60^0) - 3(\cos^2 45^0 - \sin^2 90^0) + 5\cos^2 90^0$

8. $\dfrac{4}{\cot^2 30^0} + \dfrac{1}{\sin^2 30^0} - 2\cos^2 45^0 - \sin^2 0^0$

9. $\dfrac{1}{\cos^2 30^0} + \dfrac{1}{\sin^3 30^0} - \dfrac{1}{2}\tan^2 45^0 - 8\sin^2 90^0$

10. $\cot^2 30^0 - 2\cos^2 30^0 - \dfrac{3}{4}\sec^2 45^0 + \dfrac{1}{4}\cos ec^2 30^0$

11. $(\sin^2 30^0 + 4\cot^2 45^0 - \sec^2 60^0)(\cos ec^2 45^0 \sec^2 30^0)$

12. In right triangle ABC, $\angle B = 90^0$, AB = 3cm and AC = 6cm. Find $\angle C$ and $\angle A$.

13. If $A = 30^0$, verify that:

(i) $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$ (ii) $\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A}$ (iii) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

14. If $A = 45^0$, verify that

(i) $\sin 2A = 2\sin A \cos A$ (ii) $\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

15. Using the formula, $\cos A = \sqrt{\dfrac{1+\cos 2A}{2}}$, find the value of $\cos 30^0$, it being given that $\cos 60^0 = \dfrac{1}{2}$

16. Using the formula, $\sin A = \sqrt{\dfrac{1-\cos 2A}{2}}$, find the value of $\sin 30^0$, it being given that $\cos 60^0 = \dfrac{1}{2}$

17. Using the formula, $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$, find the value of $\tan 60^0$, it being given that $\tan 30^0 = \dfrac{1}{\sqrt{3}}$.

18. If $\sin(A - B) = \dfrac{1}{2}$ and $\cos(A + B) = \dfrac{1}{2}$, then find the value of A and B.

19. If $\sin(A + B) = 1$ and $\cos(A - B) = 1$, then find the value of A and B.

20. If $\tan(A - B) = \dfrac{1}{\sqrt{3}}$ and $\tan(A + B) = \sqrt{3}$, then find the value of A and B.

21. If $\cos(A - B) = \dfrac{\sqrt{3}}{2}$ and $\sin(A + B) = 1$, then find the value of A and B.

22. If A and B are acute angles such that $\tan A = \dfrac{1}{3}$, $\tan B = \dfrac{1}{2}$ and $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$, show that $A + B = 45^0$.

23. If $A = B = 45^0$, verify that:

a) $\sin(A + B) = \sin A\cos B + \cos A\sin B$
b) $\sin(A - B) = \sin A\cos B - \cos A\sin B$
c) $\cos(A + B) = \cos A\cos B - \sin A\sin B$
d) $\cos(A - B) = \cos A\cos B + \sin A\sin B$
e) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
f) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

24. If $A = 60^0$ and $B = 30^0$, verify that:

a) $\sin(A + B) = \sin A\cos B + \cos A\sin B$
b) $\sin(A - B) = \sin A\cos B - \cos A\sin B$
c) $\cos(A + B) = \cos A\cos B - \sin A\sin B$
d) $\cos(A - B) = \cos A\cos B + \sin A\sin B$
e) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
f) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

25. Evaluate:

$$\frac{\sin^2 45^0 + \frac{3}{4}\cos ec^2 30^0 - \cos 60^0 + \tan^2 60^0}{\sin^2 30^0 + \cos^2 60^0 + \frac{1}{2}\sec^2 45^0}$$

PRACTICE QUESTIONS
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T – RATIOS OF COMPLEMENTARY ANGLES

1. Evaluate: $\cot\theta \tan(90^0 - \theta) - \sec(90^0 - \theta)\,\text{cosec}\theta + (\sin^2 25^0 + \sin^2 65^0) + \sqrt{3}(\tan 5^0 . \tan 15^0 . \tan 30^0 . \tan 75^0 . \tan 85^0)$.

2. Evaluate without using tables: $\dfrac{\sec\theta \cos ec(90^0 - \theta) - \tan\theta \cot(90^0 - \theta) + (\sin^2 35^0 + \sin^2 55^0)}{\tan 10^0 \tan 20^0 \tan 45^0 \tan 70^0 \tan 80^0}$

3. Evaluate: $\dfrac{\sec^2 54^0 - \cot^2 36^0}{\cos ec^2 57^0 - \tan^2 33^0} + 2\sin^2 38^0 \sec^2 52^0 - \sin^2 45^0$.

4. Express $\sin 67^0 + \cos 75^0$ in terms of trigonometric ratios of angles between 0^0 and 45^0.

5. If $\sin 4A = \cos(A - 20^0)$, where A is an acute angle, find the value of A.

6. If A, B and C are the interior angles of triangle ABC, prove that $\tan\left(\dfrac{B+C}{2}\right) = \cot\dfrac{A}{2}$

7. If A, B, C are interior angles of a ΔABC, then show that $\cos\left(\dfrac{B+C}{2}\right) = \sin\dfrac{A}{2}$.

8. If A, B, C are interior angles of a ΔABC, then show that $\cos ec\left(\dfrac{A+C}{2}\right) = \sec\dfrac{B}{2}$.

9. If A, B, C are interior angles of a ΔABC, then show that $\cot\left(\dfrac{B+A}{2}\right) = \tan\dfrac{C}{2}$.

10. Without using trigonometric tables, find the value of $\dfrac{\cos 70^0}{\sin 20^0} + \cos 57^0 \cos ec 33^0 - 2\cos 60^0$.

11. If $\sec 4A = \text{cosec}(A - 20^0)$, where 4A is an acute angle, find the value of A.

12. If $\tan 2A = \cot(A - 40^0)$, where 2A is an acute angle, find the value of A.

13. Evaluate $\tan 10^0 \tan 15^0 \tan 75^0 \tan 80^0$

14. Evaluate: $\left[\dfrac{\sin^2 22^0 + \sin^2 68^0}{\cos^2 22^0 + \cos^2 68^0} + \sin^2 63^0 + \cos 63^0 \sin 27^0\right]$

15. Express $\tan 60^0 + \cos 46^0$ in terms of trigonometric ratios of angles between 0^0 and 45^0.

16. Express $\sec 51^0 + \text{cosec}25^0$ in terms of trigonometric ratios of angles between 0^0 and 45^0.

17. Express $\cot 77^0 + \sin 54^0$ in terms of trigonometric ratios of angles between 0^0 and 45^0.

18. If $\tan 3A = \cot(3A - 60^0)$, where 3A is an acute angle, find the value of A.

19. If $\sin 2A = \cos(A + 36^0)$, where 2A is an acute angle, find the value of A.

20. If $\text{cosec}A = \sec(A - 10^0)$, where A is an acute angle, find the value of A.

21. If $\sin 5\theta = \cos 4\theta$, where 5θ and 4θ are acute angles, find the value of θ.

22. If $\tan 2A = \cot (A - 18^0)$, where $2A$ is an acute angle, find the value of A.

23. If $\tan 2\theta = \cot(\theta + 6^0)$, where 2θ and $\theta + 6^0$ are acute angles, find the value of θ.

24. Evaluate:

$$\frac{2\sin 68^0}{\cos 22^0} - \frac{2\cot 15^0}{5\tan 75^0} - \frac{3\tan 45^0 \tan 20^0 \tan 40^0 \tan 50^0 \tan 70^0}{5}$$

25. Evaluate:

$$\frac{\cos(90^0 - \theta)\sec(90^0 - \theta)\tan\theta}{\cos ec(90^0 - \theta)\sin(90^0 - \theta)\cot(90^0 - \theta)} + \frac{\tan(90^0 - \theta)}{\cot\theta} + 2$$

26. Evaluate:

$$\frac{\sin 18^0}{\cos 72^0} + \sqrt{3}\left\{\tan 10^0 \tan 30^0 \tan 40^0 \tan 50^0 \tan 80^0\right\}$$

27. Evaluate:

$$\frac{3\cos 55^0}{7\sin 35^0} - \frac{4(\cos 70^0 \cos ec 20^0)}{7(\tan 5^0 \tan 25^0 \tan 45^0 \tan 65^0 \tan 85^0)}$$

28. Evaluate:

$$\cos(40^0 - \theta) - \sin(50^0 + \theta) + \frac{\cos^2 40^0 + \cos^2 50^0}{\sin^2 40^0 + \sin^2 50^0}$$

29. If $A + B = 90^0$, prove that $\sqrt{\frac{\tan A \tan B + \tan A \cot B}{\sin A \sec B} - \frac{\sin^2 B}{\cos^2 A}} = \tan A$

30. If $\cos 2\theta = \sin 4\theta$, where 2θ and 4θ are acute angles, find the value of θ.

PRACTICE QUESTIONS
CLASS X: CHAPTER - 8
INTRODUCTION TO TRIGONOMETRY
TRIGONOMETRIC IDENTITIES

1. Prove that $\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=\sin\theta+\cos\theta$.

2. Prove that $\frac{1}{2}\left\{\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}\right\}=\frac{1}{\sin\theta}$.

3. Prove that: $\frac{\tan^3\alpha}{1+\tan^2\alpha}+\frac{\cot^3\alpha}{1+\cot^2\alpha}=\sec\alpha\,\mathrm{cosec}\alpha-2\sin\alpha\cos\alpha$

4. Prove that: $\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}=1+\tan A+\cot A=1+\sec A\,\mathrm{cosec}A$.

5. Prove that:. $\frac{1+\sin A-\cos A}{1+\sin A+\cos A}=\sqrt{\frac{1-\cos A}{1+\cos A}}$

6. Prove that $(\tan A+\mathrm{cosec}B)^2-(\cot B-\sec A)^2=2\tan A\cot B(\mathrm{cosec}A+\sec B)$.

7. Prove that: $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$.

8. Prove that: $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\mathrm{cosec}A+\cot A$.

9. Prove that: $\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}=\frac{2}{\sin^2 A-\cos^2 A}=\frac{2}{2\sin^2 A-1}=\frac{2}{1-2\cos^2 A}$.

10. Prove that $\frac{\sin A}{\cot A+\mathrm{cosec}A}=2+\frac{\sin A}{\cot A-\mathrm{cosec}A}$.

11. Prove that $\sqrt{\frac{\sec A-1}{\sec A+1}}+\sqrt{\frac{\sec A+1}{\sec A-1}}=2\,\mathrm{cosec}A$.

12. Prove that: $\frac{1}{\mathrm{cosec}A-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\mathrm{cosec}A+\cot A}$.

13. Prove that: $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\sec\theta+\tan\theta=\frac{1+\sin\theta}{\cos\theta}$

14. If x = $a\sin\theta+b\cos\theta$ and y = $a\cos\theta+b\sin\theta$, prove that $x^2+y^2=a^2+b^2$.

15. If $\sec\theta+\tan\theta=m$, show that $\left(\frac{m^2-1}{m^2+1}\right)=\sin\theta$.

16. Prove that: $\frac{1+\cos\theta+\sin\theta}{1+\cos\theta-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$.

17. Prove that $\sec^4 A(1-\sin^4 A)-2\tan^2 A=1$

18. If $\cos ec\theta-\sin\theta=m$ and $\sec\theta-\cos\theta=n$, prove that $\left(m^2n\right)^{2/3}+\left(mn^2\right)^{2/3}=1$

19. If $\tan\theta+\sin\theta=m$ and $\tan\theta-\sin\theta=n$, show that $m^2-n^2=4\sqrt{mn}$

20. If $a\cos\theta-b\sin\theta=c$, prove that $\left(a\sin\theta+b\cos\theta\right)=\pm\sqrt{a^2+b^2-c^2}$

21. If $\cos\theta+\sin\theta=\sqrt{2}\cos\theta$, prove that $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$

22. If $\left(\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta\right)=1$ and $\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)=1$, prove that $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=2$

23. If $\left(\tan\theta+\sin\theta\right)=m$ and $\left(\tan\theta-\sin\theta\right)=n$ prove that $\left(m^2-n^2\right)^2=16mn$

24. If $\cos ec\theta-\sin\theta=a^3$ and $\sec\theta-\cos\theta=b^3$, prove that $a^2b^2(a^2+b^2)=1$

25. If $a\cos^3\theta+3a\sin^2\theta\cos\theta=m$ and $a\sin^3\theta+3a\sin\theta\cos^2\theta=n$, prove that $\left(m+n\right)^{2/3}+(m-n)^{2/3}=2a^{2/3}$

26. Prove that $\sqrt{\sec^2\theta+\cos ec^2\theta}=\tan\theta+\cot\theta$.

27. Prove the identity: $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$.

28. Prove the identity: $\sec^6\theta=\tan^6\theta+3\tan^2\theta.\sec^2\theta+1$.

29. Prove the identity: $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2=7+\tan^2 A+\cot^2 A$.

30. If $x\sin^3\theta+y\cos^3\theta=\sin\theta\cos\theta$ and $x\sin\theta=y\cos\theta$, prove that $x^2+y^2=1$.

31. If $\sec\theta=x+\frac{1}{4x}$, Prove that $\sec\theta+\tan\theta=2x$ or $\frac{1}{2x}$.

32. Prove that $\left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right)=\frac{1}{\sin^2 A-\sin^4 A}$.

33. If $\cot\theta+\tan\theta=x$ and $\sec\theta-\cos\theta=y$, prove that $\left(x^2y\right)^{2/3}-\left(xy^2\right)^{2/3}=1$.

34. If $\frac{\cos\alpha}{\cos\beta}=m$ *and* $\frac{\cos\alpha}{\sin\beta}=n$, show that $(m^2+n^2)\cos^2\beta=n^2$.

35. If $\operatorname{cosec}\theta-\sin\theta=a$ and $\sec\theta-\cos\theta=b$, prove that $a^2b^2(a^2+b^2+3)=1$

36. If $x = r\sin A\cos C$, $y = r\sin A\sin C$ and $z = r\cos A$, prove that $r^2 = x^2 + y^2 + z^2$.

37. If $\tan A = n \tan B$ and $\sin A = m\sin B$, prove that $\cos^2 A = \frac{m^2 - 1}{n^2 - 1}$.

38. If $\sin\theta + \sin^2\theta = 1$, find the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta + 2\cos^4\theta + 2\cos^2\theta - 2$..

39. Prove that: $(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta)$

40. If $\sin + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$.

41. If $a\sec\theta + b\tan\theta + c = 0$ and $p\sec\theta + q\tan\theta + r = 0$, prove that $(br - qc)^2 - (pc - ar)^2 = (aq - bp)^2$.

42. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, then prove that $\cos^6\vartheta - 4\cos^4\theta + 8\cos^2\theta = 4$.

43. If $\tan^2\theta = 1 - a^2$, prove that $\sec\theta + \tan^3\theta\operatorname{cosec}\theta = (2 - a^2)^{3/2}$.

44. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $x^2 - y^2 = a^2 - b^2$.

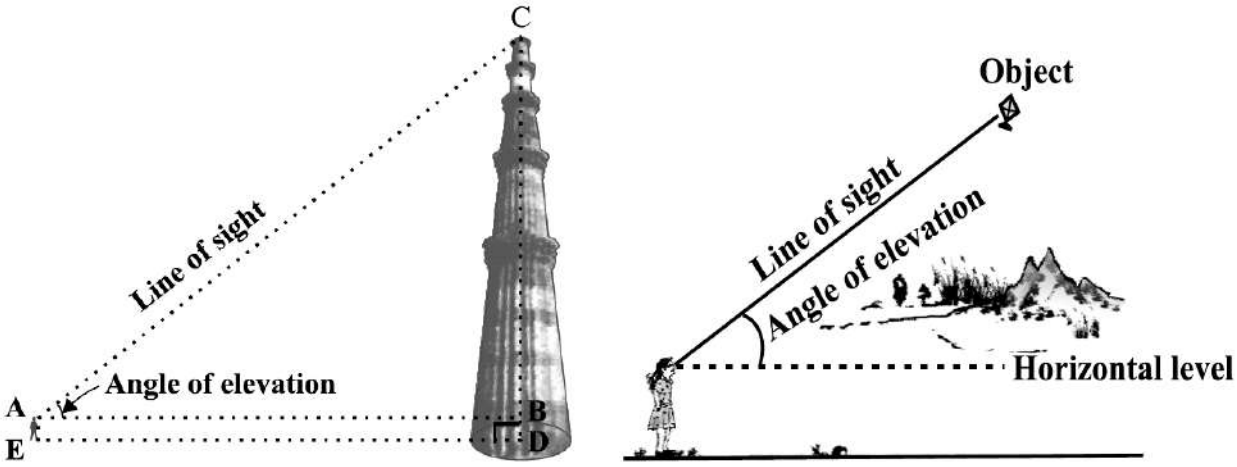
45. If $3\sin\theta + 5\cos\theta = 5$, prove that $5\sin\theta - 3\cos\theta = \pm 3$.

CLASS X : CHAPTER - 9
SOME APPLICATIONS TO TRIGONOMETRY

IMPORTANT FORMULAS & CONCEPTS

ANGLE OF ELEVATION

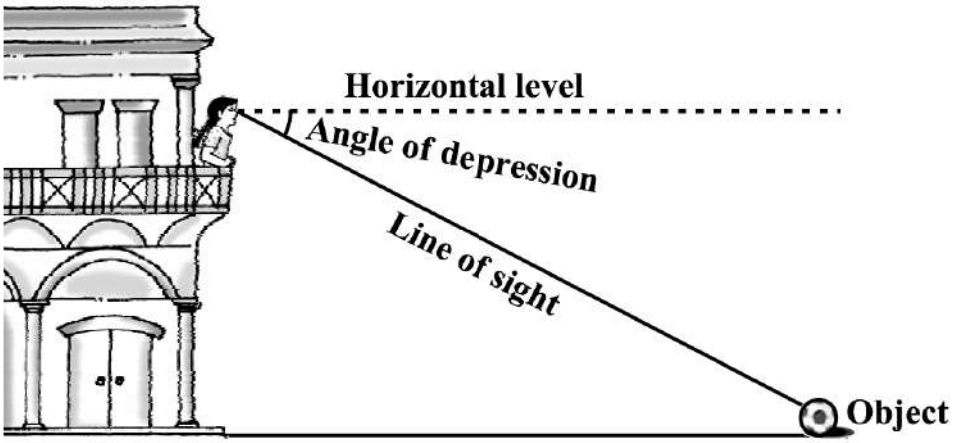
In the below figure, the line AC drawn from the eye of the student to the top of the minar is called the *line of sight*. The student is looking at the top of the minar. The angle BAC, so formed by the line of sight with the horizontal, is called the *angle of elevation* of the top of the minar from the eye of the student. Thus, the **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.



The **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object

ANGLE OF DEPRESSION

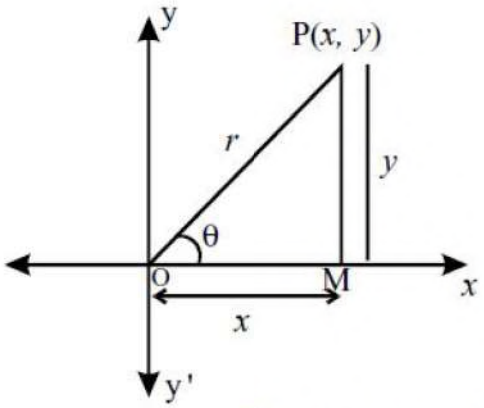
In the below figure, the girl sitting on the balcony is *looking down* at a flower pot placed on a stair of the temple. In this case, the line of sight is *below* the horizontal level. The angle so formed by the line of sight with the horizontal is called the *angle of depression*. Thus, the **angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed



Trigonometric Ratios (T - Ratios) of an acute angle of a right triangle

In XOY-plane, let a revolving line OP starting from OX, trace out $\angle XOP=\theta$. From P (x, y)draw PM $\perp$ to OX.

In right angled triangle OMP. OM = x (Adjacent side); PM = y (opposite side); OP = r (hypotenuse).



$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{y}{r}, \quad \cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{x}{r}, \quad \tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{y}{x}$$

$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{r}{y}, \quad \sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{r}{x}, \quad \cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{x}{y}$$

Reciprocal Relations

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}, \ \sec\theta = \frac{1}{\cos\theta} \ \textbf{and} \ \cot\theta = \frac{1}{\tan\theta}$$

Quotient Relations

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \text{ and } \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Trigonometric ratios of Complementary angles.

$\sin(90 - \theta) = \cos\theta$ $\quad$ $\cos(90 - \theta) = \sin\theta$

$\tan(90 - \theta) = \cot\theta$ $\quad$ $\cot(90 - \theta) = \tan\theta$

$\sec(90 - \theta) = \operatorname{cosec}\theta$ $\quad$ $\operatorname{cosec}(90 - \theta) = \sec\theta$.

Trigonometric ratios for angle of measure.

0^0, 30^0, 45^0, 60^0 and 90^0 in tabular form.

$\angle A$	0^0	30^0	45^0	60^0	90^0
sinA	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosA	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanA	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosecA	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
secA	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cotA	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

MCQ WORKSHEET-I
CLASS X: CHAPTER – 9
SOME APPLICATIONS TO TRIGONOMETRY

1. The angle of elevation of the top of a tower from a point on the ground, which is 20m away from the foot of the tower is 60^0. Find the height of the tower.
(a) $10\sqrt{3}$ m (b) $30\sqrt{3}$ m (c) $20\sqrt{3}$ m (d) none of these

2. The height of a tower is 10m. What is the length of its shadow when Sun's altitude is 45^0?
(a) 10 m (b) 30 m (c) 20 m (d) none of these

3. The angle of elevation of a ladder leaning against a wall is 60^0 and the foot of the ladder is 9.5 m away from the wall. Find the length of the ladder.
(a) 10 m (b) 19 m (c) 20 m (d) none of these

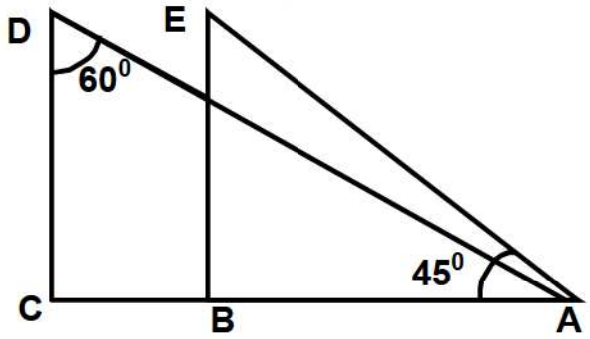
4. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3}$: 1, what is the angle of elevation of the Sun?
(a) 30^0 (b) 60^0 (c) 45^0 (d) none of these

5. What is the angle of elevation of the Sun when the length of the shadow of a vertical pole is equal to its height?
(a) 30^0 (b) 60^0 (c) 45^0 (d) none of these

6. From a point on the ground, 20 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is 60^0, what is the height of the tower?
(a) $10\sqrt{3}$ m (b) $30\sqrt{3}$ m (c) $20\sqrt{3}$ m (d) none of these

7. If the angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary, find the height of the tower.
(a) 10 m (b) 6 m (c) 8 m (d) none of these

8. In the below fig. what are the angles of depression from the observing positions D and E of the object A?
(a) 30^0, 45^0 (b) 60^0 , 45^0 (c) 45^0, 60^0 (d) none of these



9. The ratio of the length of a rod and its shadow is 1: $\sqrt{3}$. The angle of elevation of the sun is
(a) 30^0 (b) 60^0 (c) 45^0 (d) none of these

10. If the angle of elevation of a tower from a distance of 100m from its foot is 60^0, then the height of the tower is
(a) $100\sqrt{3}$ m (b) $\frac{200}{\sqrt{3}}$ m (c) $50\sqrt{3}$ m (d) $\frac{100}{\sqrt{3}}$ m

MCQ WORKSHEET-II
CLASS X: CHAPTER – 9
SOME APPLICATIONS TO TRIGONOMETRY

1. If the altitude of the sun is at 60^0, then the height of the vertical tower that will cast a shadow of length 30m is

(a) $30\sqrt{3}$ m (b) 15 m (c) $\frac{30}{\sqrt{3}}$ m (d) $15\sqrt{2}$ m

2. A tower subtends an angle of 30^0 at a point on the same level as its foot. At a second point 'h' metres above the first, the depression of the foot of the tower is 60^0. The height of the tower is

(a) $\frac{h}{2}$ m (b) $\frac{h}{3}$ m (c) $\sqrt{3}h$ m (d) $\frac{h}{\sqrt{3}}$ m

3. A tower is $100\sqrt{3}$ m high. Find the angle of elevation if its top from a point 100 m away from its foot.

(a) 30^0 (b) 60^0 (c) 45^0 (d) none of these

4. The angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of the tower is 30^0. Find the height of the tower.

(a) $10\sqrt{3}$ m (b) $30\sqrt{3}$ m (c) $20\sqrt{3}$ m (d) none of these

5. The string of a kite is 100m long and it makes an angle of 60^0 with the horizontal. Find the height of the kite, assuming that there is no slack in the string.

(a) $100\sqrt{3}$ m (b) $\frac{200}{\sqrt{3}}$ m (c) $50\sqrt{3}$ m (d) $\frac{100}{\sqrt{3}}$ m

6. A kite is flying at a height of 60m above the ground. The inclination of the string with the ground is 60^0. Find the length of the string, assuming that there is no slack in the string.

(a) $40\sqrt{3}$ m (b) $30\sqrt{3}$ m (c) $20\sqrt{3}$ m (d) none of these

7. A circus artist is climbing a 20m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30^0.

(a) 10 m (b) 30 m d) none of these

8. A tower is 50m high, Its shadow ix ': :n the sun's altitude is 45^0 than when it is 30^0. Find the value of 'x'

(a) $100\sqrt{3}$ m (b) $\frac{200}{\sqrt{3}}$ m d) none of these

9. Find the angular elevation of the sun when the shadow of a 10m long pole is $10\sqrt{3}$ m.

(a) 30^0 (b) 60^0 (c) 45^0 (d) none of these

10. A vertical pole stands on the level ground. From a point on the ground 25m away from the foot of the pole, the angle of elevation of its top is found to be 60^0. Find the height of the pole.

(a) $25\sqrt{3}$ m (b) $\frac{25}{\sqrt{3}}$ m (c) $50\sqrt{3}$ m (d) none of these

MCQ WORKSHEET-III
CLASS X: CHAPTER – 9
SOME APPLICATIONS TO TRIGONOMETRY

1. A kite is flying at a height of 75m above the ground. The inclination of the string with the ground is 60^0. Find the length of the string, assuming that there is no slack in the string.
(a) $40\sqrt{3}$ m (b) $30\sqrt{3}$ m (c) $50\sqrt{3}$ m (d) none of these

2. The angle of elevation of the tope of a tree from a point A on the ground is 60^0. On walking 20m away from its base, to a point B, the angle of elevation changes to 30^0. Find the height of the tree.
(a) $10\sqrt{3}$ m (b) $30\sqrt{3}$ m (c) $20\sqrt{3}$ m (d) none of these

3. A 1.5m tall boy stands at a distance of 2m from lamp post and casts a shadow of 4.5m on the ground. Find the height of the lamp post.
(a) 3 m (b) 2.5 m (c) 5 m (d) none of these

4. The height of the tower is 100m. When the angle of elevation of the sun changes from 30^0 to 45^0, the shadow of the tower becomes 'x' meters less. The value of 'x' is
(a) $100\sqrt{3}$ m (b) 100 m (c) $100(\sqrt{3}-1)$ m (d) $\frac{100}{\sqrt{3}}$

5. The tops of two poles of height 20m and 14m are connected by a wire. If the wire makes an angle of 30^0 with horizontal, then the length of the wire is
(a) 12 m (b) 10 m (c) 8 m (d) 6 m

6. If the angles of elevation of a tower from two points distant a and b (a > b) from its foot and in the same straight line from it are 30^0 and 60^0, then the height of the tower is
(a) $\sqrt{a+b}$ m (b) $\sqrt{a-b}$ m (c) $\sqrt{ab}$ m (d) $\sqrt{\frac{a}{b}}$ m

7. The angles of elevation of the top of a tower from two points at a distance of 'a' m and 'b' m from the base of the tower and in the same straight line with it are complementary, then the height of the tower is
(a) $\sqrt{a+b}$ m (b) $\sqrt{a-b}$ m (c) $\sqrt{ab}$ m (d) $\sqrt{\frac{a}{b}}$ m

8. From the top of a cliff 25m high the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is
(a) 25 m (b) 50 m (c) 75 m (d) 100 m

9. If the angle of elevation of a cloud from a point 200m above a lake is 30^0 and the angle of depression of its reflection in the lake is 60^0, then the height of the cloud above the lake is
(a) 200 m (b) 500 m (c) 30 m (d) 400 m

10. The angle of elevation of a cloud from a point 'h' meter above a lake is 'α'. The angle of depression of its reflection in the lake is 45^0. The height of the cloud is
(a) h.tanα (b) $\frac{h(1+\tan\alpha)}{(1-\tan\alpha)}$ (c) $\frac{h(1-\tan\alpha)}{(1+\tan\alpha)}$ (d) none of these

PRACTICE QUESTIONS
CLASS X: CHAPTER – 9
SOME APPLICATIONS TO TRIGONOMETRY

1. A vertical stick 10 cm long casts a shadow 8 cm long. At the same time, a tower casts a shadow 30 m long. Determine the height of the tower.

2. An observer, 1.5 m tall, is 28.5 m away from a tower 30 m high. Find the angle of elevation of the top of the tower from his eye.

3. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60^0. When he retreats 20m from the bank, he finds the angle to be 30^0. Find the height of the tree and the breadth of the river.

4. A boy is standing on ground and flying a kite with 150m of string at an elevation of 30^0. Another boy is standing on the roof of a 25m high building and flying a kite at an elevation of 45^0. Find the length of string required by the second boy so that the two kites just meet, if both the boys are on opposite side of the kites.

5. An aeroplane flying horizontally 1000m above the ground, is observed at an angle of elevation 60^0 from a point on the ground. After a flight of 10 seconds, the angle of elevation at the point of observation changes to 30^0. Find the speed of the plane in m/s.

6. An aeroplane when flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angles of the elevation of the two planes from the same point on the ground are 60^0 and 45^0 respectively. Find the vertical distance between the aeroplanes at that instant.

7. An aeroplane at an altitude of 200 m observes the angles of depression of opposite points on the two banks of a river to be 45^0 and 60^0. Find the width of the river.

8. The shadow of a flag staff is three times as long as the shadow of the flag staff when the sun rays meet the ground at an angle of 600. Find the angle between the sun rays and the ground at the time of longer shadow.

9. A vertically straight tree, 15m high is broken by the wind in such a way that it top just touches the ground and makes an angle of 60^0 with the ground, at what height from the ground did the tree break?

10. A man in a boat rowing away from lighthouse 100 m high takes 2 minutes to changes the angle of elevation of the top of lighthouse from 60^0 to 45^0. Find the speed of the boat.

11. As observed from the top of a light house, 100m above sea level, the angle of depression of ship, sailing directly towards it, changes from 30^0 to 45^0. Determine the distance travelled by the ship during the period of observation.

12. A man standing on the deck of ship, which is 10m above the water level, observes the angle of elevation of the top of a hill as 60^0 and the angle of depression of the base of the hill as 30^0. Calculate the distance of the hill from the ship and the height of the hill.

13. The angles of elevation of the top of a tower from two points at a distance of 'a' m and 'b' m from the base of the tower and in the same straight line with it are complementary, then prove that the height of the tower is $\sqrt{ab}$

14. A tower stands vertically on the ground. From a point on the ground, which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower.

15. An electrician has to repair an electric fault on a pole of height 5 m. She needs to reach a point 1.3m below the top of the pole to undertake the repair work. What should be the length of the ladder that she should use which, when inclined at an angle of 60° to the horizontal, would enable her to reach the required position? Also, how far from the foot of the pole should she place the foot of the ladder? (You may take $\sqrt{3} = 1.73$)

16. An observer 1.5 m tall is 28.5 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°. What is the height of the chimney?

17. From a point P on the ground the angle of elevation of the top of a 10 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 45°. Find the length of the flagstaff and the distance of the building from the point P. (You may take $\sqrt{3} = 1.73$)

18. The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°. Find the height of the tower.

19. The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

20. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

21. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

22. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

23. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

24. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

25. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.

26. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

27. A man on cliff observes a boat an angle of depression of 30^0 which is approaching the shore to the point immediately beneath the observer with a uniform speed. Six minutes later, the angle of depression of the boat is found to be 60^0. Find the time taken by the boat to reach the shore.

28. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

29. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

30. A tree is broken by the storm. The top of the tree touches the ground making an angle 30° and at a distance of 30 m from the root. Find the height of the tree.

31. A tree 12m high, is broken by the storm. The top of the tree touches the ground making an angle 60°. At what height from the bottom the tree is broken by the storm.

32. At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$. In walking 192 m towards the tower, the tangent of the angle of elevation is $\frac{3}{4}$. Find the height of the tower.

33. The pilot of an aircraft flying horizontally at a speed of 1200km/hr, observes that the angle of depression of a point on the ground changes from 30^0 to 45^0 in 15 seconds. Find the height at which the aircraft is flying.

34. If the angle of elevation of the cloud from a point h m above a lake is A and the angle of depression of its reflection in the lake is B, prove that the height of the cloud is $\frac{h(\tan B + \tan A)}{(\tan B - \tan A)}$

35. The angle of elevation of cloud from a point 120m above a lake is 30^0 and the angle of depression of the reflection of the cloud in the lake is 60^0. Find the height of the cloud.

36. The angle of elevation of cloud from a point 60m above a lake is 30^0 and the angle of depression of the reflection of the cloud in the lake is 60^0. Find the height of the cloud.

37. The angle of elevation of a jet plane from a point A on the ground is 60^0. After a flight of 15 seconds, the angle of elevation changes to 30^0. If the jet plane is flying at a constant height of $1500\sqrt{3}$ m, find the speed of the jet plane.

38. The angle of elevation of a jet plane from a point A on the ground is 60^0. After a flight of 30 seconds, the angle of elevation changes to 30^0. If the jet plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed of the jet plane.

39. There are two temples, one on each bank of river, just opposite to each other. One temple is 50m high. From the top of this temple, the angles of depression of the top and foot of the other temple are 30^0 and 60^0 respectively. Find the width of the river and the height of other temple.

40. A ladder rests against a wall at an angle α to the horizontal, its foot is pulled away from the wall through a distant **a**, so that it slides a distance **b** down the wall making an angle β with the horizontal. Show that $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$.

41. From a window, h meter above the ground of a house in a street , the angle of elevation and depression of the top and the foot of another house on the opposite side of the street are θ and ϕ respectively. Show that the height of the opposite house is h (1 + tanθcotϕ).

42. From a window, 15 meters high above the ground of a house in a street , the angle of elevation and depression of the top and the foot of another house on the opposite side of the street are 30^0 and 45^0 respectively. Find the height of the opposite house.

43. Two stations due south of a leaning tower which leans towards the north are at distances a and b from its foot. If α and β are the elevations of the top of the tower from these stations, prove that its inclination θ to the horizontal is given by $\cot\theta = \frac{b\cot\alpha - a\cot\beta}{b-a}$.

44. The angle of elevation of a cliff from a fixed point is θ. After going up a distance of 'k'meters towards the top of the cliff at an angle of ϕ, it is found that the angle of elevation is α. Show that the height of the cliff is $\frac{k(\cos\phi - \sin\phi.\cot\alpha)}{\cot\theta - \cot\alpha}$.

45. A round balloon of radius **r** subtends an angle α at the eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is $r\sin\beta.\text{cosec}\frac{\alpha}{2}$

46. The angle of elevation of the top of a tower from a point on the same level as the foot of the tower is α. On advancing 'p' meters towards the foot of the tower the angle of elevation becomes β. Show that the height 'h' of the tower is given by $h = \left(\frac{p\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}\right)$m. Also determine the height of the tower if p = 150° m, α = 30° and β = 60°.

47. From the top of a light- house the angle of depression of two ships on the opposite sides of it are observed to be α and β. If the height of the light-house be 'h' meter and the line joining the ships passes through the foot of the light house, show that the distance between the ships is $h\left(\frac{\tan\alpha + \tan\beta}{\tan\alpha.\tan\beta}\right)$ meters.

48. An electrician has to repair on electric fault on a pole of height 4m. she needs to reach a point 1.3m below the top of the pole to undertake the repair work. What should be the height of the ladder that she should use at angle of 60° to the horizontal, would enable her reach the required position? Also, how far the foot of the pole should she place the foot of the ladder.(take $\sqrt{3}$ = 1.732)

49. The angle of elevation of a jet fighter from a point A on the ground is 60°. After a flight of 15 sec, the angle of elevation changes to 30o. If the jet is flying at a speed of 720 km/hr, find the constant height at which the jet is flying.

50. A man on a top of a tower observes a truck at angle of depression α where $\tan\alpha = \frac{1}{\sqrt{5}}$ and sees that it is moving towards the base of the tower. Ten minutes later, the angle of depression of truck found to be β where $\tan\beta = \sqrt{5}$ if the truck is moving at uniform speed determine how much more time it will take to reach the base of the tower.

51. At the foot of a mountain the elevation of its summit is 45^0; after ascending 1000m towards the mountain up a slope of 30^0 inclination, the elevation is found to be 60^0. Find the height of the mountain.

52. If the angle of elevation of cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake be β, prove that the distance of the cloud from the point of observation is $\frac{2h\sec\alpha}{\tan\beta-\tan\alpha}$.

53. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height 'h'. At a point on the plane, the angles of elevation of the bottom and top of the flag staff are α and β respectively. Prove that the height of the tower is $\frac{h\tan\alpha}{\tan\beta-\tan\alpha}$.

54. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30^0 to 45^0, how soon after this, will the car reach the tower? Give your answer to the nearest second.

55. Two pillars of equal height and on either side of a road, which is 100m wide. The angles of depression of the top of the pillars are 60^0 and 30^0 at a point on the road between the pillars. Find the position of the point between the pillars and the height of the tower.

56. The angle of elevation of the top of a tower from a point A due north of the tower is α and from B due west of the tower is β. If AB = d, show that the height of the tower is $\frac{d\sin\alpha\sin\beta}{\sqrt{\sin^2\alpha-\sin^2\beta}}$.

57. The angle of elevation of the top of a tower from a point A due south of the tower is α and from B due east of the tower is β. If AB = d, show that the height of the tower is $\frac{d}{\sqrt{\cot^2\alpha+\cot^2\beta}}$.

58. From an aeroplane vertically above a straight horizontal road, the angles of depression of two consecutive milestones on opposite sides of the aeroplane are observed to be α and β. Show that the height in miles of aeroplane above the road is given by $\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$.

59. A tree standing on horizontal plane is leaning towards east. At two points situated at distances a and b exactly due west on it, angles of elevation of the top are respectively α and β. Prove that the height of the top from the ground is $\frac{(b-a)\tan\alpha\tan\beta}{\tan\alpha-\tan\beta}$.

60. The length of the shadow of a tower standing on level plane is found to be 2x metres longer when the sun's altitude is 30^0 than when it was 45^0. Prove that the height of tower is $x\left(\sqrt{3}+1\right)m$.