CLASS-11
SUBJECT-PHYSICS
Chapter 6: GRAVITATION

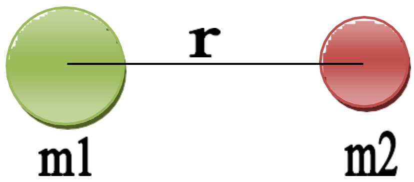
Newton's Law of Gravitation: According to this law any two body in the universe having mass attract each other with a force which is directly proportional to the product of the masses inversely proportional to the square of the distance between them.

Let two masses be m_1 and m_2 and r being the distance between them. Then according to the Newton's Law of gravitation,

$F \propto m_1 m_2$

And, $F \propto 1 / r^2$

Therefore, $F \propto (m_1 m_2) / r^2$



$F = (G m_1 m_2) / r^2$

where G is the constant of proportionality and is called **universal gravitational constant**. Value of $\mathbf{G = 6.67 \times 10^{-11}\ Nm^2/kg^2}$ (SI System)

$G = 6.67 \times 10^{-8}$ dyne cm^2/g^2 (CGS System)

Dimensional formula of G = $[M^{-1}L^3T^{-2}]$

Evidence of Newton's Law of Gravitation:

1. The earth and other planets go round the sun by virtue of this force.
2. Tides in the sea are result of gravitational force acting on sea water by sun and moon.
3. The time calculated for the launch of satellites is on the basis of gravitational force. If it would have been wrong satellites could not be launched.
4. The prediction of solar and lunar eclipses is done on the basis of gravitational law accurately.

Vector form of Newton's Law of Gravitation: Let us take two bodies having masses m_1 and m_2 and r being the distance between them.

Let $\mathbf{F_{12}}$ be the force exerted on m_1 by m_2 and $\mathbf{F_{21}}$ be the force exerted on m_2 by m_1.

Let $\mathbf{r_{21}}$ be the unit vector pointing from m_2 to m_1.

Similarly $\mathbf{r_{12}}$ be the unit vector pointing from m_1 to m_2.

$\mathbf{F_{12}} = [(G m_1 m_2) / r^2]\, \mathbf{r_{21}}$

And, $\mathbf{F_{21}} = [(G m_1 m_2) / r^2]\, \mathbf{r_{12}}$

Since, $\mathbf{r}_{21} = - \mathbf{r}_{12}$

Therefore, $\mathbf{F}_{12} = - \mathbf{F}_{21}$

Therefore these two forces are equal and opposite of each other. Hence, this law is in accordance with the Newton's third law of motion. Hence the two make action reaction pair.

⁜**Gravitation**: Force of attraction between any two bodies in the universe.

⁜**Gravity**: If any body having mass is replaced by earth or a celestial body having a comparable mass then gravitation is termed as gravity. OR ◎ Gravity is the force with which earth attracts any body towards itself i.e. weight.

Weight is the measure of gravity.

⁜**Characteristics** of force of gravity:

1. Gravity follows inverse square law as the force of gravitation is inversely proportional to the square of distance between two objects.
2. Gravitational forced is a central force i.e. it acts along a line joining the centre of two bodies.
3. It is a conservative force i.e. it does not depend upon the path followed by the two objects.
4. Gravitational force is directly proportional to the product of the two masses of the objects between which the force acts.

⁜**Acceleration due to gravity (g)**: Gravity is a force and hence some acceleration would be produced due to it. The acceleration so produced is called acceleration due to gravity.

The acceleration experienced by an object falling freely only under the action of gravity is called acceleration due of gravity.

Hence on the surface of earth, $\mathbf{g = F / m = 9.8\ m/s^2}$ where 'F' is the gravitational force and 'm' is the mass of the object.

SI unit ◎$\mathbf{m/s^2}$ and Dimensions ◎$\mathbf{[M^0L^1T^{-2}]}$

⁜**Relationship between g and G:** Consider earth to be a perfect sphere of mass 'M' and radius 'R'. Consider a body of mass 'm' be placed on its surface.

According to Newton's universal law of gravitation, $F = GMm / R^2$ --- (1)

Also force acting due to gravity = $F = mg$ --- (2)

Equating the above two equations we get, $\mathbf{g = GM / R^2}$

Therefore ‘g’ does not depend upon mass of the body but depends only on the mass and radius of the planet. Hence, all objects big or small, light or heavy fall at the same time from the same height. Lighter objects fall slower than heavier objects due to the air resistance experienced by them.

✳Factors affecting ‘g’:

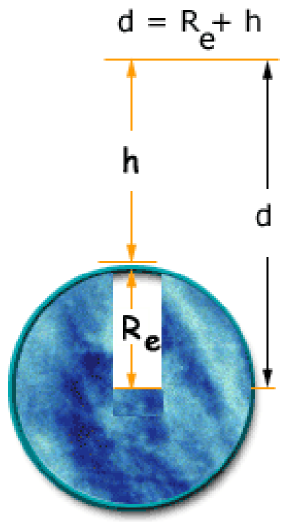
1. Shape of earth ◎ Earth is not a perfect sphere but bulges at equator. Therefore if a body is taken from pole to equator its distance from the centre of the earth will change. Consequently, the gravitational force also varies.

2. Altitude / Height from the surface of earth ◎ Let **g** be the value of acceleration due to gravity at the surface of earth and **g'** at a height **h** above the surface of earth.
   If the earth is considered as a sphere of homogeneous composition, then **g** at any point on the surface of the earth is given by:

$$g = \frac{G M_e}{R_e^2} \longrightarrow \text{(i)}$$

Now we will determine the value of **g** at a distance ($R_e + h$) from the centre of the earth.
In this condition distance is ($R_e + h$)
The acceleration due to gravity is:

$$g' = \frac{G M_e}{(R_e + h)^2} \longrightarrow \text{(ii)}$$



dividing equation (ii) by (i)

$$\frac{g'}{g} = \frac{\dfrac{\cancel{G M_e}}{(R_e + h)^2}}{\dfrac{\cancel{G M_e}}{R_e^2}}$$

$$\frac{g'}{g} = \frac{R_e^2}{(R_e + h)^2}$$

$$\frac{g'}{g} = R_e^2 (R_e + h)^{-2}$$

Taking R_e^{-2} common

$$\frac{g'}{g} = \cancel{R_e^2}\,\cancel{R_e^{-2}}\left(1 + \frac{h}{R_e}\right)^{-2}$$

$$\frac{g'}{g} = \left(1 + \frac{h}{R_e}\right)^{-2}$$

Expanding by binomial theorem and
neglecting higher powers of h/R_e

$$\boxed{\frac{g'}{g} = \left(1 - 2\frac{h}{R_e}\right)}$$

This expression indicates that the acceleration due to gravity decreases with altitude.

3. Variation with depth © Let **g** be the value of acceleration due to gravity at the surface of earth and **g'** at a depth **d** below the surface of earth.
   If the earth is considered as a sphere of homogeneous composition of density **ρ**, then **g** at any point on the surface of the earth is given by:

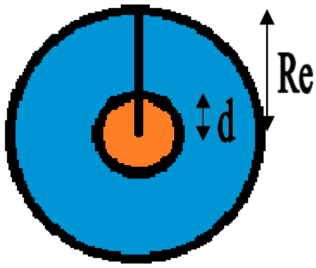
   $$g = G M_e / R_e^2 = G\, 4\, \pi\, R_e^3\, \rho / 3 R_e^2 = 4\, G\, \pi\, R_e\, \rho\ / 3 \qquad --- \qquad (1)$$

Only the part of earth below the point will contribute gravitational force. Therefore,

$\mathbf{g'} = G M' / (R_e - d)^2$

$\Rightarrow \mathbf{g'} = G\, 4\, \pi\, (R_e - d)^3\, \rho / 3\, (R_e - d)^2$

$\Rightarrow \mathbf{g'} = 4\, G\, \pi\, (R_e - d)\, \rho / 3 \qquad --- \qquad (2)$



Dividing equation (2) by (1) we get,

$\mathbf{g' / g} = (Re - d) / Re = \mathbf{1 - (d / Re)}$

Since $[1 - (d / Re)] < 1 \Rightarrow \mathbf{g' < g}$

This expression indicates that the acceleration due to gravity decreases with depth.

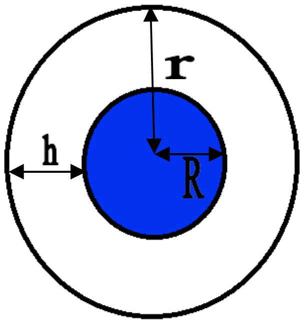
At the centre of earth $d = Re \Rightarrow \mathbf{g' = 0}$

4. Variation with Latitude and Earth's Rotation © The expression for variation of g with rotation of earth and latitude is given as: $\mathbf{g' = g\,[1 - (R_e\, \omega^2 \cos^2 \lambda) / g]}$ where ω stands for angular velocity of earth's rotation and λ stands for latitude at a point which is the angle made by the line joining the point and the centre of the earth makes with the equatorial plane.
   (i) As per the relation we can see that acceleration due to gravity decreases with increase in the angular velocity of rotation of earth.

(ii) Also if λ increases then $\cos \lambda$ decreases increasing the value of **g'.** Therefore, acceleration due to gravity increases with increase in latitude.
At poles, $\lambda = 90°$ and $\cos 90° = 0 \Rightarrow$ **$g' = g$**
At equator, $\lambda = 0°$ and $\cos 0° = 1 \Rightarrow$ **$g' = g - R_e\,\omega^2$** which means there is more effect of rotation at equator than at poles.

Principle of Launching of satellite: Let earth be a perfect sphere of mass 'M' and radius 'R'. If a tower is constructed whose height is more than the atmospheric height and a body be projected with a horizontal velocity 'v' then it reaches a point P on the surface of earth. If we keep increasing the velocity then a velocity will be reached such that the body does not fall on the surface of earth instead it starts orbiting around the earth. Then this velocity is called orbital velocity. This is the principle of launching of a satellite where a satellite is launched from a great height using a launch vehicle i.e. rocket with orbital velocity.

Orbital Velocity: Orbital velocity is the minimum velocity given to a body with which the body starts orbiting around the earth in a circular path.
Let 'm' be the mass of satellite and 'M' is the mass of earth which is considered to be a sphere of radius 'R' with centre 'O' and 'r' be the radius of the orbit. Let 'h' be the height of the orbit above the surface of earth i.e. $r = R + h$



Centripetal force required for the satellite to go around the circular path is given by, $F_c = mv_o^2 / r$ --- (1)
This force is provided by gravitational force between satellite and planet given by Newton's law of gravitation i.e. $F = G\,M\,m / r^2$ --- (2)
Equating (1) and (2), we get

$$mv_o^2 / r = G\,M\,m / r^2$$

$$\Rightarrow v_o^2 = G\,M / r$$

$$\Rightarrow \mathbf{v_o = \sqrt{G\,M / r}}$$

Also $G\,M = gR^2 \Rightarrow$ Equating this relation in above equation we get

$$v_o = \sqrt{(gR^2) / r} = \sqrt{g\,R^2 / (R + h)}$$

Special case: If $h << R$ then $R + h \approx R$. Therefore, $v_o = \sqrt{(g\,R^2) / R} = \sqrt{gR}$
Putting the values of $g = 9.8\ m/s^2$ and $R = 6400$ km for **earth** we get, **$v_o \approx 8$ km / sec**. Therefore the minimum orbital velocity required to launch a satellite close to the surface of earth is 8 km / sec.

Time Period of satellite: We know, time = displacement / velocity
Therefore the time period of a satellite = circumference of orbit / orbital velocity

$$\Rightarrow T = 2\,\pi\,r / v_o = 2\,\pi\,(R + h) / \sqrt{g\,R^2 / (R + h)}$$

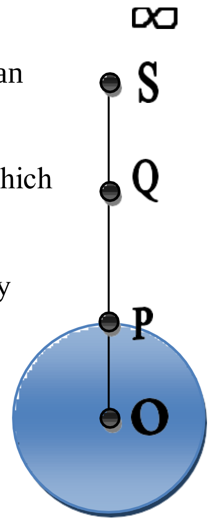
$$\Rightarrow \mathbf{T = (2\pi / R)\sqrt{(R + h)^3 / g}}$$

Special Case: If $h << R$, $T = 2\pi\sqrt{R / g}$
Putting the values for earth we get T = 108 minute.

Height of the satellite:
From the expression of time period we can get the expression of height of the satellite as

$$\mathbf{h = [(T^2R^2g) / (4\,\pi^2)]^{1/3} - R}$$

Escape Velocity: The minimum velocity of projection provided to a body so that it can escape the gravitational pull of the planet is called escape velocity.



Earth's gravitational pull exist up to infinity. So we require a velocity of projection which will project the body upwards so that it goes up to infinity.
Let us consider earth to be a perfect sphere of mass 'M', radius 'R' and centre 'O'.
Consider a body of mass 'm' on the surface of earth which is projected with a velocity 'v_e'. Consider three points P, Q and S such that OP = R, OQ = x and QS = dx.
Therefore OS = x + dx
Gravitational force of attraction between body at Q and earth = $F = GM\,m / x^2$
Now let the body get displaced to S by a distance dx. Then dw is the small work done in displacing the body from Q to S.
Therefore, $dw = Fdx = GMm\, dx / x^2$
Let W be the total work done in displacing a body from surface of earth to infinity. Then W is calculated by integrating dw from surface of earth to infinity.

$W = \int dw = \int GMm\, dx / x^2$ [from x = R to x = infinity]

$\Rightarrow$ $W = GMm \int dx / x^2$ [from x = R to x = infinity]

$\Rightarrow$ $W = GM\,m\,[\,x^{-1} / -1\,]$ [from x = R to x = infinity]

$\Rightarrow$ $W = -\,G\,M\,m\,[\,1/\infty - 1/R\,]$

$\Rightarrow$ $W = G\,M\,m / R$ --- (1)

This work is done by kinetic energy imparted to the body at the surface of earth.
Therefore, $\frac{1}{2} m\, v_e^{\,2}$ = kinetic energy = W

$\Rightarrow$ $\frac{1}{2} m\, v_e^2 = G\,M\,m / R$

$\Rightarrow$ $\mathbf{v_e = \sqrt{(2GM/R)}}$ --- (2)

Now $g = GM / R^2$ Substituting this value in equation (2) we get,

$\mathbf{v_e = \sqrt{2\,g\,R}}$ --- (3)

Hence escape velocity depends mass and radius
Substituting the values for earth we get,

$\mathbf{v_e \approx 11.2}$ **km/sec**

Practically a body should be projected with little more velocity than 11.2 km/sec as air resistance has not been taken into account.

Why moon is not having any atmosphere of its own?
Gravity of moon < < gravity of earth. Therefore, the escape velocity of moon < < escape velocity of earth. Now if the root mean square velocity of gases > escape velocity of moon then they all escape the surface of moon and hence it has no atmosphere.
For similar reasons light gases on earth have also escaped as rms velocity of light gases > escape velocity of earth.

Types of satellites: There are two types of satellites ⓪ geostationary satellites and polar satellites.

1. **Geostationary satellites:** Satellites which have the same time period and same sense of rotation as that of earth about its own axis. With respect to any point on earth the position of geostationary satellites never change and always remain stationary.

   Characteristics of a Geostationary Satellite:

   (i) The time period of such satellites is 24 hours.

   (ii) They have the same sense of rotation as that of earth i.e. west to east.

   (iii) The orbit of the satellite is concentric and coplanar with the equatorial plane of earth.

   (iv) The height of the orbit from the surface of earth is 36000 km.

   Applications: They are used as communication satellites and for weather reporting.

2. **Polar satellites:** The satellites which revolve in the polar orbits around the earth are called polar satellites. Polar orbit is an orbit whose angle of inclination with equatorial plane is 90°.

   Characteristics of a Polar Satellite:

   (i) Polar satellites have a low altitude and cross any location on earth many times in a day.

   (ii) The height of the orbit is about 500 km to 800 km from the surface of earth.

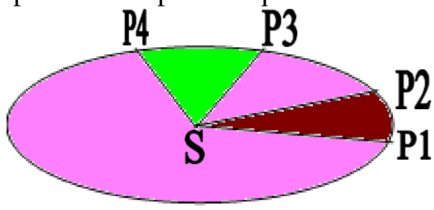
   (iii) The time period of polar satellites is around 100 minutes.

   Applications: They are used as communication satellites, forecasting weather, in studying upper region of atmosphere, to determine exact shape and dimension of earth and to study the cosmic rays and solar radiations.

✳ **Kepler's Law of Planetary Motion:** There are three laws of planetary motion given by Kepler.

(i) **Law of orbits:** It states that every planet revolves around sun in an elliptical orbit with sun at one of its focus.

(ii) **Law of areas:** It states that the radius vector drawn from sun to planet sweeps out equal areas in equal intervals of time.



⇒ Area P_1SP_2 = Area P_3SP_4

⇒ $P_1P_2 < P_3P_4$

Dividing by time,

⇒ $P_1P_2 / t < P_3P_4 / t$

⇒ Velocity of planet at P_1 < Velocity of planet at P_3

Therefore **the linear velocity of planet increases when planet comes closer to the sun**. This law follows from the law of conservation of angular momentum.

(iii) **Law of periods:** It states that square of time period of revolution of a planet around sun is directly proportional to cube of semi-major axis of elliptical orbit.

⇒ $\mathbf{T^2 \propto r^3}$

✳ **Derivation of Newton's Law of Gravitation from Kepler's Law:** Let the time period be 'T' and angular velocity 'ω', mass of planet 'M', radius of planet 'R' and radius of orbit 'r'.

Now, $\omega = 2\pi / T$

But, $T^2 \propto r^3$ => $T^2 = Kr^3$

The centripetal force acting on the planet, $F = M r \omega^2$

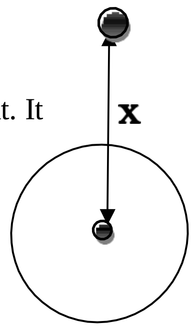
- $F = M\,r\,4\pi^2 / T^2$
- $F = M\,r\,4\pi^2 / (Kr^3)$
- $\mathbf{F = (4\pi^2 / K)\,(M / r^2)}$

  Now to get Newton's formula of gravitational force $4\pi^2 / K$ is directly proportional to M.
- $4\pi^2 / K = GM$
- $\mathbf{F = G\,M\,m / r^2}$

Thus we can arrive at Newton's formula of gravitation from Kepler's law.

Gravitational Field: It is the modified area around a material body / mass in which its influence can be felt due to the gravitational force it exerts i.e. when a mass is kept near another mass it experiences a gravitational force of attraction which becomes zero when the other mass is taken at a distance infinitely away from the first one.

Gravitational field extends up to infinity.



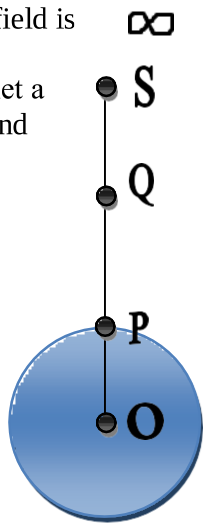
Gravitational Field Intensity (I): Gravitational field intensity at a point in the gravitational field is defined as the force experienced by a unit mass placed at that point. It is represented by **I** and is a vector quantity. **SI unit:** newton / kg = N/kg and **CGS unit:** dyne / g

Let us consider a body of unit mass at a distance 'x' from a planet of mass 'M' and radius 'R'.

Force experienced by unit mass placed at P = GM / x^2 --- [Since m = 1]

- $\mathbf{I = GM / x^2}$
- $\mathbf{I = F / m}$

Gravitational Potential: Gravitational potential at a point inside the gravitational field is the amount of work done in bringing a unit mass from infinity to that point.



Consider earth to be a perfect sphere of mass 'M', centre 'O' and radius 'R'. Also let a unit mass be placed at point P on surface of earth. Therefore OP = R. Let OQ = x and OS = x + dx

Force experienced by unit mass placed at point Q = GM / x^2

If the mass is further displaced through a distance dx then the small work done is

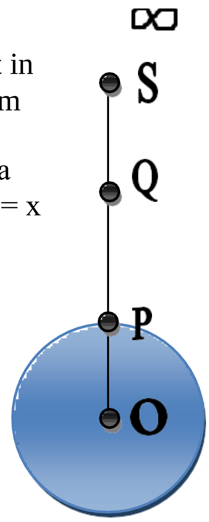
$dw = F\,dx = G\,M\,dx / x^2$

Work done in bringing body from infinity to P is calculated by integrating it.

- $W = \int dw$ within limits $x = \infty$ to $x = R$
- $W = \int G\,M\,dx / x^2$ within limits $x = \infty$ to $x = R$
- $W = G\,M \int x^{-2}\,dx$ within limits $x = \infty$ to $x = R$
- $W = -\,G\,M\,[1/x]$ within limits $x = \infty$ to $x = R$
- $W = -\,G\,M\,[1/R - 1/\infty]$
- $\mathbf{W = -\,G\,M / R}$ **= Gravitational Potential**

Potential is negative when force is attractive in nature and when it is positive the force is repulsive in nature. Here the potential is negative implying gravitational force is attractive.

Also gravitational potential increases as we move away from earth and the maximum value of gravitational potential is zero when $r = \infty$.



Gravitational Potential Energy: Gravitational potential energy of a body is the point in the gravitational field is defined as the amount of work done in bringing that body from infinity to that point.

Consider earth to be a perfect sphere of mass 'M', centre 'O' and radius 'R'. Also let a body of mass 'm' be placed at point P on surface of earth. Therefore OP = R. Let OQ = x and OS = x + dx

Force experienced by mass 'm' placed at point Q = $G M m / x^2$

Let dw be the small work done in displacing the body from Q to S by a distance dx.

Then $dw = G M m\, dx / x^2$

- $W = \int dw$ from $x = \infty$ to $x = R$
- $W = \int G M m\, dx / x^2$ from $x = \infty$ to $x = R$
- $U = - G M m [1/x]$ from $x = \infty$ to $x = R$
- $U = - G M m [1/\infty - 1/R]$
- $U = - G M m / R$
- **Gravitational potential energy = gravitational potential × mass of the body**

Gravitational potential energy is negative showing that gravitational force is attractive in nature.

(i) If body is brought from distance r_2 to r_1 such that $r_2 > r_1$

Gravitational potential energy = $- G M m [1/r_1 - 1/r_2]$

Since $r_2 > r_1 \Rightarrow 1/r_2 < 1/r_1$ => Change in gravitational potential energy is negative

- As the body is moved closer to earth gravitational potential energy decreases.

(ii) If $r_1 = R$ and $r_2 = R + h$

If we are going from r_1 to r_2 then the change in potential energy = $- G M m [1/r_2 - 1/r_1]$

- $\Delta U = - G M m [1/(R + h) - 1/R]$
- $\Delta U = (- G M m / R) [(1 + h/R)^{-1} - 1]$

Since h << R therefore expanding it binomially,

$\Delta U = (-G M m / R) [1 - h/R - 1 + \text{negligible terms}]$

- $\boldsymbol{\Delta U = G M m h / R^2}$
- $\boldsymbol{\Delta U = m g h}$ [Since $g = G M / R^2$]
- **Gravitational potential energy = m g h**