CHAPTER - 13

LIMITS AND DERIVATIVES

KEY POINTS

- $\lim_{x \to c} f(x) = l$ if and only if
- $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = l$
- $\lim_{x \to c} \alpha = \alpha$, where α is a fixed real number.
- $\lim_{x \to c} x^n = c^n$, for all $n \in N$
- $\lim_{x \to c} f(x) = f(c)$, where f(x) is a real polynomial in x.

Algebra of limits

Let f, g be two functions such that $\lim_{x \to c} f(x) = l$ and $\lim_{x \to c} g(x) = m$, then

- $\lim_{x \to c} [\alpha\, f(x)] = \alpha \lim_{x \to c} f(x)$

  $= \alpha\, l$ for all $\alpha \in R$

- $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = l \pm m$

- $\lim_{x \to c} [f(x).g(x)] = \lim_{x \to c} f(x). \lim_{x \to c} g(x) = lm$

- $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{l}{m}$, $m \neq 0$ $g(x) \neq 0$

- $\lim_{x \to c} \frac{1}{f(x)} = \frac{1}{\lim_{x \to c} f(x)} = \frac{1}{l}$ provided $l \neq 0$ $f(x) \neq 0$

- $\lim_{x \to c} [(f(x)]^n = \left[\left(\lim_{x \to c} f(x)\right)\right]^n = l^n$, for all $n \in N$

Some important theorems on limits

- $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(-x)$

- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

- $\lim_{x \to 0} \frac{\sin x}{x} = 1$ where x is measured in radians.

- $\lim_{x \to 0} \frac{\tan x}{x} = 1 \left[\text{Note that } \lim_{x \to 0} \frac{\cos x}{x} \neq 1\right]$

- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

- $$\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$$
- $$\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$$
- $$\lim_{x \to 0} (1 + x)^{1/x} = e$$

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{d(\cot x)}{dx} = -\cos ec^2 x$$

$$\frac{d(\sec x)}{dx} = \sec x \tan x$$

$$\frac{d(\cos ec\, x)}{dx} = -\cos ec\, x.\cot x$$

$$\frac{d(x^n)}{dx} = n.x^{n-1}$$

$$\frac{d\left(e^x\right)}{dx}=e^x \qquad \frac{d\left(a^x\right)}{dx}=a^x.\log a$$

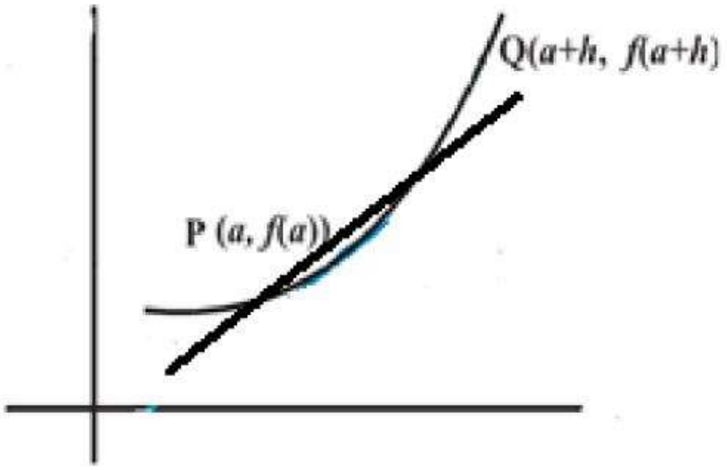
$$\frac{d\left(\log_e x\right)}{dx}=\frac{1}{x}$$

$$\frac{d\left(constant\right)}{dx}=0$$

Laws of Logarithm

- $\log_e A+\log_e B=\log_e\left(AB\right)$
- $\log_e A-\log_e B=\log_e\left(\frac{A}{B}\right)$
- $\log_e A^m=m\log_e A$
- $\log_a 1=0$
- If $\log_B A=x$ then $B^x=A$

Let y = f(x) be a function defined in some neighbourhood of the point ‘a’. Let P(a, f(a)) and Q(a + h, f (a + h)) are two points on the graph of f(x) where h is very small and $h\neq 0$.



Slope of PQ = $\frac{f(a+h)-f(a)}{h}$

If $h \to 0$, point Q approaches to P and the line PQ becomes a tangent to the curve at point P.

$\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ (if exists) is called derivative of f(x) at the point 'a'. It is denoted by f'(a).

Algebra of derivatives

- $\frac{d}{dx}(cf(x)) = c.\frac{d}{dx}(f(x))$ where c is a constant
- $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$
- $\frac{d}{dx}(f(x).g(x)) = f(x).\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$
- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{d}{dx}(f(x)) - f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$
- If y = f(x) is a given curve then slope of the tangent to the curve at the point (h,k) is given by $\left.\frac{dy}{dx}\right]_{(h,k)}$ and is denoted by 'm'.