CLASS- IX
UNIT -14
MATHEMATICAL REASONING

READING MATERIAL PART-2

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Mathematical Reasoning Part-2

Topics covered:

5.QUANTIFIERS- “THERE EXISTS”,”FOR ALL”.

6.IMPLICATIONS- “IF-THEN”,”ONLY IF,” ”IF AND ONLY IF”

7.CONTRAPOSITIVE AND CONVERSE OF A STATEMENT

8.VALIDATING MATHEMATICAL STATEMENTS

(a) By direct method
(b) By the method of contrapositive
(c) Method of contradiction
(d)Using a counter example.

Mathematical Reasoning: Quantifiers

- **QUANTIFIERS**: **Quantifiers** are phrases like, "There exists" and "For all". Another phrase which appears in **mathematical** statements is "there exists".

- A Word closely connected with "there exists" is "for every" (or for all). Consider a statement.

We come across many mathematical statement containing these phrases.

For example – Consider the following statements

p : For every real number x, x is less than x+1.
q : There exists a triangle whose all sides are equal

IMPLICATIONS : “if-then”, “only if” and “if and only if”

If -then: Sufficient condition

EX: If a number is a multiple of 9,then it is a multiple of 3.
p: a number is a multiple of 9
q: a number is a multiple of 3
P implies q is denoted by p => q.
P is a sufficient condition for q.

Only if: Necessary condition

EX: A number is a multiple of 9 only if it is a multiple of 3
P:a number is a multiple of 9
q: a number is a multiple of 3.
q is a necessary condition for p

“if and only if “If and only if is bi conditional:

Both are true or false together
Both sufficient and necessary condition.

Example: A rectangle is a square if and only if all its four sides are equal..

CONTRAPOSITIVE AND CONVERSE OF A STATEMENT:

Statement: if p then q

Converse: if q then p
Contrapositive: if not q then not p

Example:1
Statement: If a tringle is equilateral, it is isosceles.

Contrapositive of this statement is " If a triangle is not isosceles, then it is not equilateral'.
Contra positive of a given statement "if p, then q" is if ~q, then ~p
If statement is true then contra positive is also true.

Example:2

The converse of a given statement "if p, then q" is "if q, then p".

Statement: If N is divisible by 6, then N is divisible by 3

Converse:
N is divisible by 3, then N is divisible by 6

Validating Statements (if -then)

In order to prove the statement "if p then q" we need to show that any one of the

following case is true.
Case 1 By assuming that p is true, prove that q must be true.(Direct method)
Case 2 By assuming that q is false, prove that p must be false.(Contrapositive Method)

Validating Statements (if and only if)

In order to prove the statement "p if and only if q", we need to show.

If p is true, then q is true and
If q is true, then p is true

Validate by contradiction

We assume that p is not true i.e. ~p is true. Then, we arrive at some result which contradicts our assumption. Therefore, we conclude that p is true.

Validity of statements

Validity of a statement means checking when the statement is true and when it is not true

Validity of statement with "If-then"

To show statement r : "If p then q is true", we can adopt the following methods:

(a) Direct method : Assume p is true and show q is true, i.e., $p \Rightarrow q$.

(b) Contrapositive method : Assume ~ q is true and show ~ p is true, i.e., $\sim q \Rightarrow \sim p$.

(c) Contradiction method : Assume that p is true and q is false and obtain a contradiction from assumption.

(d) By giving a counter example : To prove the given statement r is false we give a counter example.

Consider the following statement.
"r : All prime numbers are odd". Now the statement 'r' is false as 2 is a prime number and it is an even number.

Question 1:

Show that the statement p: "If x is a real number such that $x^3 + 4x = 0$, then x is 0" is true by
(i) direct method (ii) method of contradiction (iii) method of contra positive

Answer:
p: "If x is a real number such that $x^3 + 4x = 0$, then x is 0".
Let q: x is a real number such that $x^3 + 4x = 0$
r: x is 0.

(i) To show that statement p is true, we assume that q is true and then show that r is true.
Therefore, let statement q be true.
So, $x^3 + 4x = 0$
=> $x(x^2 + 4) = 0$
=> $x = 0$ or $x^2 + 4 = 0$
However, since x is real, it is 0.
Thus, statement r is true.
Therefore, the given statement is true.

(ii) To show statement p to be true by contradiction, we assume that p is not true.
Let x be a real number such that $x^3 + 4x = 0$ and let x is not 0.
Therefore, $x^3 + 4x = 0$
=> $x(x^2 + 4) = 0$
=> $x = 0$ or $x^2 + 4 = 0$

=> $x(x^2 + 4) = 0$
=> $x = 0$ or $x^2 + 4 = 0$
=> $x = 0$ or $x^2 = -4$
However, x is real. Therefore, $x = 0$, which is a contradiction since we have assumed that x is not 0.
Thus, the given statement p is true

(iii) To prove statement p to be true by contra positive method, we assume that r is false and prove that q must be false.
Here, r is false implies that it is required to consider the negation of statement r.
This obtains the following statement:
~r: x is not 0.
It can be seen that $(x^2 + 4)$ will always be positive.
$x \neq 0$ implies that the product of any positive real number with x is not zero.
Let us consider the product of x with $(x^2 + 4)$.
=> $x(x^2 + 4) \neq 0$
=> $x^3 + 4x \neq 0$
This shows that statement q is not true.
Thus, it has been proved that
$\sim r \Rightarrow \sim q$
Therefore, the given statement p is true

Converse, Inverse, and Contrapositive

- If you <u>interchange</u> the antecedent and the consequent of a conditional, you form a new conditional known as the ***converse*** of the original statement.
- If you <u>negate</u> both the antecedent and the consequent, you form the ***inverse*** of the statement.
- If you <u>interchange and negate</u> the antecedent and consequent, you form the ***contrapositive*** of the statement.

Related Conditional Statements		
Conditional (original)	$p \rightarrow q$	If p, then q.
Converse	$q \rightarrow p$	If q, then p.
Inverse	$\sim p \rightarrow \sim q$	If not p, then not q.
Contrapositive	$\sim q \rightarrow \sim p$	If not q, then not p.