Key Points
Chapter-16_Probability_ XI(mathematics)

An experiment is said to be a random experiment if there are more than one possible outcomes and whose outcomes cannot be predicted in advance.
Ex: Tossing of a coin, throwing of a die, drawing of a card from playing cards…….
Coin: On tossing a coin there are two possibilities either head may come up or tail may come up.
Die: A die is a well-balanced cube with its six faces marked with numbers (dots) from 1 to 6, one number on the one face. The plural of die is dice.
Cards: A pack of cards consists of four suits i.e.., Diamonds, Hearts, Spades and Clubs. Each suit consists of 13 cards, nine cards numbered 2, 3, 4, ……, 10 and an Ace, a King, a Queen and a Jack or Knave. Colour of Spades and Clubs is black and that of Hearts and Diamonds is red. King, Queen and Jack cards are called Face cards.
All possible results of a random experiment are called its outcomes.
Sample Space: The set of all possible outcomes of a random experiment is called the Sample Space and is denoted by 'S'.
All elements of a sample space are known as Sample points.
An event is a subset of a sample space associated with a random experiment.
Types of Events:
Simple or Elementary event: Each outcome of a random experiment is called an elementary event (Or) An event contains single element is called a simple event.
Compound Event: If an event has more than one outcome is called compound event.
sure event: An event associated with a random experiment is called *Sure event*, if it always occurs when the random experiment is performed.
Impossible event: An event associated with a random experiment is an *impossible event* if it never occurs.
Ex: On throwing of a die, the sample space is S = {1, 2, 3, 4, 5, 6}

{1}, {2}, {3}, {4}, {5}, {6} are called elementary events.
Getting an even prime number is an event A. A = {2} is an elementary event.
Getting an odd number is an event B. B = {1, 3, 5} is a compound event
Getting a number > 4 is an event C. C = {5, 6} is a compound event.
Getting a number < 7 is an event D. D = { 1,2,3,4,5,6 } = s is sure event.
Getting a number > 6 is an event E. E = { } = Φ is an impossible event.

Equally Likely Events: Two events are said to be equally likely, if none of them is expected to occur in preference to the other.
For example, if we toss a coin, each outcome head or tail is equally likely to occur.
Algebra of Events:
Complementary events: Given an event A, the complement of A is the event consisting of all sample space outcomes that do not correspond to the occurrence of A and is written as Not A Or $\bar{A}$ Or A^1 Or A^c.
Ex: On throwing of a die, the sample space is S = {1, 2, 3, 4, 5, 6}
A = Getting an odd number = {1, 3, 5} ; Not A = S – A = { 2, 4, 6}
The Event 'A or B': If A and B are any two events of a random experiment, then the event A or B denoted by A U B is the event of all outcomes which are either in A or in B or in both.
Eg: In the random experiment of throwing a die once, if
A = getting a number less than 3 = { 1, 2 }
B = getting an odd number = { 1, 3, 5 } then A U B = {1, 2, 1, 3, 5} = { 1, 2, 3, 5 }.
The Event 'A and B': If A and B are any two events of a random experiment, then the event 'A and B' denoted by $A \cap B$ is the event of all outcomes which are both in A and B.
Eg: In the random experiment of throwing a die once, if
A = getting a number less than 3 = { 1, 2 }
B = getting a prime number = { 2, 3, 5 } then $A \cap B$ = { 2 }.
The Event 'A but not B': If A and B are any two events of a random experiment, then the event 'A but not B' denoted by $A \cap B^1$ is the event of all outcomes which are in A but not in B. $A \cap \bar{B} = A - B = A - (A \cap B)$.

Eg: In the random experiment of throwing a die once   S = {1, 2, 3, 4, 5, 6}
A = getting a number less than 3 = { 1, 2 }, B = getting a number greater than 4 = { 5, 6 }
$\bar{B}$ = S – B = {1, 2, 3, 4} then $A \cap \bar{B}$ = A – B = {1, 2}. $A \cap \bar{B}$ = {1, 2}.

The Event 'neither A nor B': If A and B are any two events associated with a random experiment, then the event 'neither A nor B' denoted by $\bar{A} \cap \bar{B} = \overline{A \cup B}$.

The Event 'not A or not B': $\bar{A} \cup \bar{B} = \overline{A \cap B}$.

The Event exactly one of A or B : $\bar{A} \cap B$ *or* $A \cap \bar{B}$

Mutually Exclusive Events: Two or more events are said to be *mutually exclusive events* if the occurrence of any one of the event prevents the occurrence of the other. *i.e*
Two events A and B are mutually exclusive if $A \cap B = \emptyset$. i.e. A and B are disjoint sets.
Eg: Three coins are tossed.   S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}
A = Not more than one head = {HTT, THT, TTH, TTT},
B = Exactly two heads = {HHT, HTH, THH},
C = Three heads = {HHH}
Here A∩B = B∩C = A∩C = ∅. So, the events A and B, B and C, A and C are mutually exclusive events.

Exhaustive Events: If $E_1, E_2, \ldots, E_n$ are n events of a sample space S and if $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$, then E_1, $E_2, \ldots E_3$ are called *exhaustive events.*
Two or more events are said to be *exhaustive events* if their union is the sample space.

Mutually Exclusive and Exhaustive Events:
If $E_1, E_2, \ldots E_n$ are n events of a sample space S and if $E_i \cap E_j = \Phi$ for every i ≠ j i.e. E_i and E_j are pairwise disjoint and $E_1 \cup E_2 \cup E_3 \cup \ldots \cup E_n = S$, then the events $E_1, E_2, \ldots, E_n$ are called mutually exclusive and exhaustive events.

Probability of an Event: If there are 'n' equally likely elementary events (outcomes) of a random experiment, 'm' outcomes are favourable to the event A, then the probability of happening or occurrence of the event A is defined as

$$P(A) = \frac{m}{n} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{n(A)}{n(S)}$$

Note: Clearly (i) $0 \le P(A) \le 1$, (ii) P(A) = 0, if A = ∅ (iii) P(A) = 1, if A = S

Probability of not happening an event: Let A be any event. Then $\bar{A}$ or A^c or A' denotes the Event of not happening A.
Therefore, if there are 'n' outcomes for a random experiment and 'm' outcomes are favourable to A, then (n – m) outcomes are not favourable to A.

$$\therefore P(\text{not } A) = P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A).$$

$$P(A) + P(\bar{A}) = 1$$

Addition rule for Probability: We have $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$\frac{n(A \cup B)}{n(S)} = \frac{n(A)}{n(S)} + \frac{n(B)}{n(S)} - \frac{n(A \cap B)}{n(S)}$$

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For any three events:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$

Note: Al least ⇒ ≥ (greater than or equal to); At most ⇒ ≤ (less than or equal to)
Total number of possible outcomes on throwing of
(i) 'n' coins = 2^n (ii) 'n' dice = 6^n
(iii) P (Sure event) = 1;   (iv) P(Impossible event) = 0

***Factorial* (!):**
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7 \times 6! = 7 \times 6 \times 5!$
$n! = 1 \times 2 \times 3 \times \ldots \ldots (n-1) \times n = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$
$= n \times (n-1)!$

$$n_{c_r} = \frac{n!}{(n-r)!\,r!}$$

$$7_{c_3} = \frac{7!}{(7-3)!3!} = \frac{7!}{(4)!3!} = \frac{7\times6\times5\times4\times3\times2\times1}{4\times3\times2\times1 \times3\times2\times1} = 7 \times 5 = 35 \quad \text{(Or)} \quad 7_{c_3} = \frac{7\times6\times5}{3!} = \frac{7\times6\times5}{3\times2\times1} = 35$$

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