MOTION IN A STRAIGHT LINE (study material)

(Chapter-3)

. Introduction

. Position, path length and displacement

. Average velocity and average speed

. Instantaneous velocity and speed

. Acceleration

. Kinematic equations for uniformly accelerated motion

. Relative velocity

INTRODUCTION : Motion is common to everything in the universe. We walk, run and ride a bicycle. Even when we are sleeping, air moves into and out of our lungs and blood flows in arteries and veins.

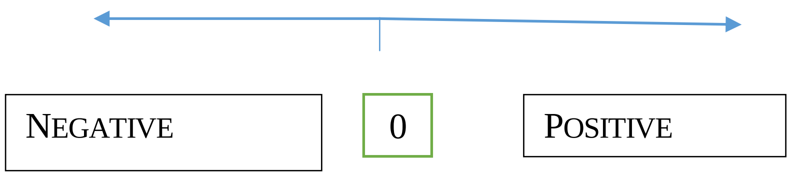
We see leaves falling from trees and water flowing down a dam. Automobiles and planes carry people from one place to the other. The earth rotates once every twenty-four hours and revolves round the sun once in a year. The sun itself is in motion in the Milky Way, which is again moving within its local group of galaxies**.**

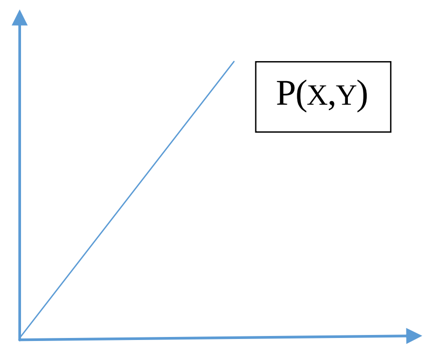
Kinematics: study of motion of body without going into the causes of motion.

Frame of reference: (or reference frame):consists of an abstract coordinate system and the set of physical reference points that uniquely fix (locate and orient) the coordinate system and standardize measurements within that frame.

It is convenient to choose a rectangular coordinate system consisting of three mutually perpendicular axes, labelled X-, Y-, and Z- axes. The point of intersection of these three axes is called origin (O) and serves as the reference point. The coordinates (x, y. z) of an object describe the position of the object with respect to this coordinate system. To measure time, we position a clock in this system. This coordinate system along with a clock constitutes a frame of reference.

Position: Location of an object in space. Position can be determined by coordinate axis that is marked in units of length and that has positive and negative directions.





At time t=0 the position of object along x-axis is (x) and along y-axis is (y).

Point object: The size of the object is much smaller than the distance it moves in a reasonable duration of time.

Rest : If position of an object remains constant with respect to its surroundings or frame of reference with the passage of time. Examples: Hills, Buildings, etc.

Motion: A body is said to be in motion if its position is changing with respect to its surroundings or frame of reference with the passage of time.

Examples: 1. Rushing of vehicles

2. earth revolving around the sun

Rest and Motion are relative terms: An object which is at rest can also be in motion simultaneously.

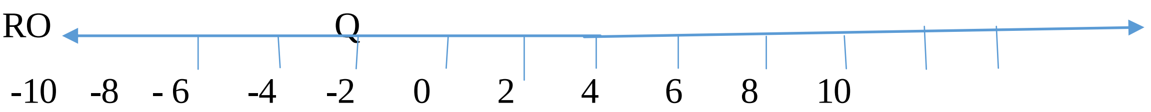
Eg. The passengers sitting in a moving train are at rest w.r.t. each other but they are in motion w.r.t. surroundings.

Motion in a straight line(One Dimensional Motion) The motion of the object is said to be one dimensional if only one of the three coordinates is required to be specified with respect to time. It is also known as rectilinear motion. In such a motion the object moves in a straight line.

Example: A bus moving on straight road

Distance: The length of actual path followed by the object is called the distance.

If an object starts from R, goes to Q and then come back to O. Then path length is the length RO+OQ+QO=(8+6+6)m. This path length is called the distance.



(negative)………………………… Length in meter……………………(positive)

It is a scalar (independent of path selection)

Displacement: The change in position of an object in a fixed direction is called its displacement. (It is always a straight line)

Displacement is directed along line segment joining the initial and final positions of a moving body.

It is a vector quantity and a shortest line between initial and final position.

If a body changes from one position x_1 to another position x_2 , then the displacement Δx in time interval $\Delta t = t_2 - t_1$, is $\Delta x = x_2 - x_1$

If an object moves from Q to R, then

Displacement=final position-initial position= (6m)- (-8m) = 14m

1.The displacement is a vector quantity.

2. The displacement has units of length.

3. The displacement of an object in a given time interval can be positive, zero or negative.

4. The actual distance travelled by an object in a given time interval can be equal to or greater than the magnitude of the displacement.

5. The displacement of an object between two points does not tell exactly how the object actually moved between those points.

6. The displacement of a particle between two points is a unique path, which can take the particle from its initial to final position.

7. The displacement of an object is not affected due to the shift in the origin of the position-axis.

Scalar quantity : a physical quantity which has magnitude only. Eg.: Length, Mass, Time, Speed, Energy, etc.

Vector quantity: a physical quantity which has both magnitude as well as direction.

Eg.: Displacement, Velocity, Acceleration, Momentum, Force, etc

Speed: rate of change of distance of a particle is called speed

Speed = Distance travelled /$TimeTaken$

1. Speed is a scalar quantity.

2. Speed is either positive or zero but never negative.

3. Speed of a running car is measured by ‘speedometer’.

4. Speed is measured in

i) cm/s (cm s^{-1}) in cgs system of units

ii) m/s (m s^{-1}) in SI system of units and

iii) km/h (km h^{-1}) in practical life when distance and time involved are large.

Uniform Speed If an object covers equal distances in equal intervals of time, then its speed is called uniform speed.

Non-uniform or Variable Speed If an object covers unequal distances in equal intervals of time, then its speed is called nonuniform or variable speed.

Average Speed:ratio of the total distance travelled by it to the total time taken.

Average speed =$Totaldistancetravelled / Totaltimetaken$

Instantaneous Speed: When a body is moving with variable speed, the speed of the body at any instant is called instantaneous speed.

Velocity : The time rate of change of displacement of a particle

Velocity = $Displacement/ timetaken$

1. Velocity is a vector quantity.

2. Direction of velocity is the same as the direction of displacement of the body.

3. Velocity can be either positive, zero or negative.

4. Velocity can be changed in two ways: (i) changing the speed of the body ii) keeping the speed constant but by changing the direction.

5. Velocity is measured in i) cm/s (cm s^{-1}) in cgs system of units

ii) m/s (m s^{-1}) in SI system of units and

iii) km/h (km h^{-1}) in practical life when distance and time involved are large.

Average velocity:ratio of the total (net) displacement covered by it to the total time taken

Average Velocity = $Netdisplacement/Totaltimetaken$

ratio of change in position or displacement (Δx) to the time intervals (Δt), in which thedisplacement occurs.

(Average velocity)$\bar{v} = x_2 - x_1 / t_2 - t_1 = \Delta x / \Delta t$

For a body moving with uniform acceleration Vav. = $u+v/ 2$

Instantaneous Velocity: When a body is moving with variable velocity, the velocity of the body at any instant is called instantaneous velocity

The velocity at a particular instant is called instantaneous velocity.

v = $\lim \Delta t \to 0 \Delta x / \Delta t$

v = dx/dt (Rate of change of position at a particular instant) (Instantaneous velocity is simply called velocity)

For uniform motion, velocity (Instantaneous velocity) is the same as the average velocity at all instants. .

<u>Acceleration</u>: If the velocity of a body changes either in magnitude or in direction or both, then it is said to have acceleration.

Average acceleration $(\overline{a})$: The change of velocity divided by the time interval.

$(\overline{a}) = v_2 - v_1 / t_2 - t_1 = \Delta v / \Delta t$

The SI unit of acceleration is m s^{-2}

1. Acceleration is a vector quantity.

2. Direction of acceleration is the same as the direction of velocity of the body.

3. Acceleration can be either positive, zero or negative.

4. Acceleration of a body is zero when it moves with uniform velocity

5 Acceleration of free fall in vacuum is uniform and is called acceleration due to gravity (g) and it is equal to 980 cms^{-2} or 9.8 ms^{-2}. If a body has uniform velocity, it has no acceleration

6. Acceleration is measured in i) cm/s^2 or (cm s^{-2}) in cgs system of units

ii) m/s2 (m s^{-2}) in SI system of units

7. A body can have zero velocity and non-zero acceleration.

Differential Calculus

Let y be a function ofx i.e. y = f (x)

Suppose value of x increases by small amount Δx then value of y also increases by Δy.

The ratio is called the average rate of change of y with respect to x.

When Δx approaches zero, the limiting value of dy/ dx

is called differential coefficients or derivative of y w.r.t. x and is denoted by dy/ dx

.

Hence $\lim_{\Delta t \to 0} \quad \Delta x/\Delta t = dx/dt$

Some important principles of Differentiation

(i) Let C be a constant then

$d/dt\ (C) = 0$ EX. $d/dt\ (2) = 0$

(ii) $d/dt\ (x^n) = n\, x^{n-1}$ EX. $d/dt\ (x^2) = 2\, x^{2-1}$

$= 2\, x$

(iii) *L*et y = u ± v where u and v are the functions of x. (v) Thendy/ dx = du/ dx ± dv/ dx

(iv) Let y = uv then = u dv/ dx + v du/ dx.

(v) d/ dx (log x) = 1/x

(vi) d/ dx (e^x) = e^x

(vii) d/ dx (sin$x$) = cos x

(xi) d/ dx (cos x) = - sin$x$

INTEGRAL CACULUS

Integration : Integration is reverse process of differentiation. It is the process of finding a function whose derivative is given. If derivative of function f (x) w. r. t. x is f1 (x) then integration of f^1 (x) w. r. t. x is f (x).

then $\int f^1 (x)dx = f (x)$

Some principles of Elementary Integrals:

i) $\int x^n dx = X^{n+1}/n+1 + C$ Where C is constant of integration

Ex. 1) $\int x^2\, dx = X^{n+1}/n+1$

$= x^3 / 3$

Ex. 2) $\int 2x\, dx = 2\ (X^{n+1}/n+1)$

$= 2\ (X^{1+1}/1+1)$

$=2\ X^2\ / 2$

$= X^2$

ii) $\int dx = x+C$

EX. $\int_a^b Xdx = [x2\ /2]^b{}_a$

$= ½\ [b^2 - a^2]$

iii) $\int dx/x = \log_e x + C$

iv) $\int \cos dx = \sin x + C$

v) $\int \sin x.\, dx = - \cos x + C$

X

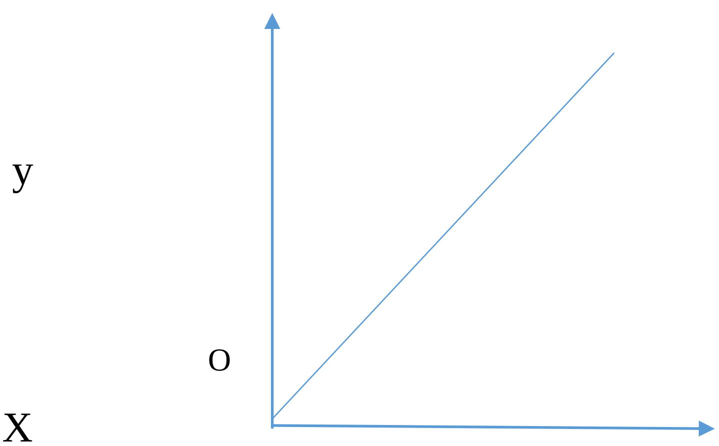
GRAPHS: MOTION IN A STRAIGHT LINE

GRAPH: A graph is a pictorial representation of motion of an object.

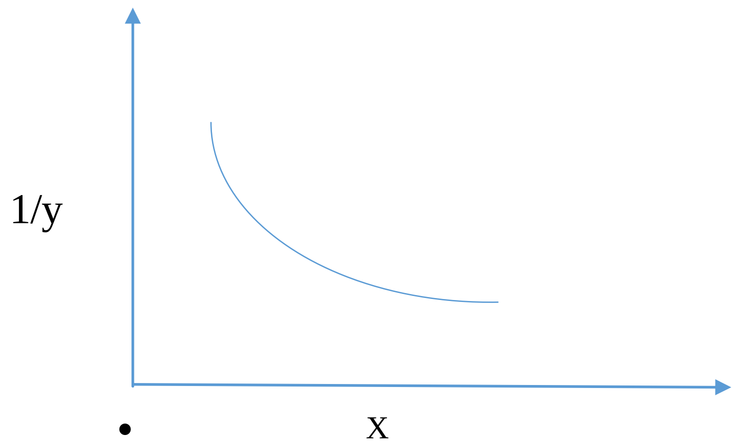
Graph is a tool to represent and analyse different aspects of motion of an object.

(i) if x ά y

• When the two variables are directly proportional to each other. • (powers of x and y are same and one only) , then the relation is called Linear equation and the graph between the two variables must be a Straight line

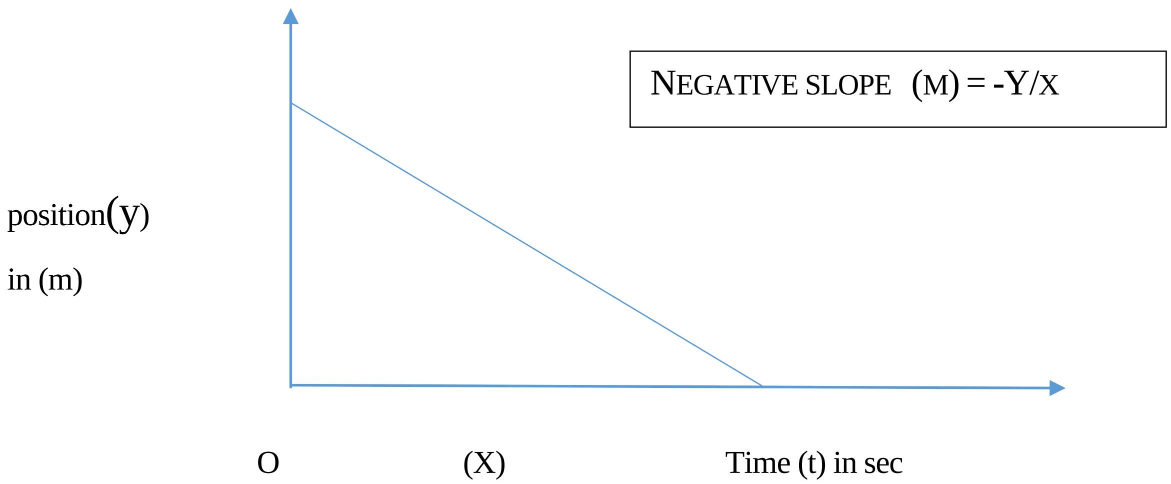


ii)if x ά 1/y

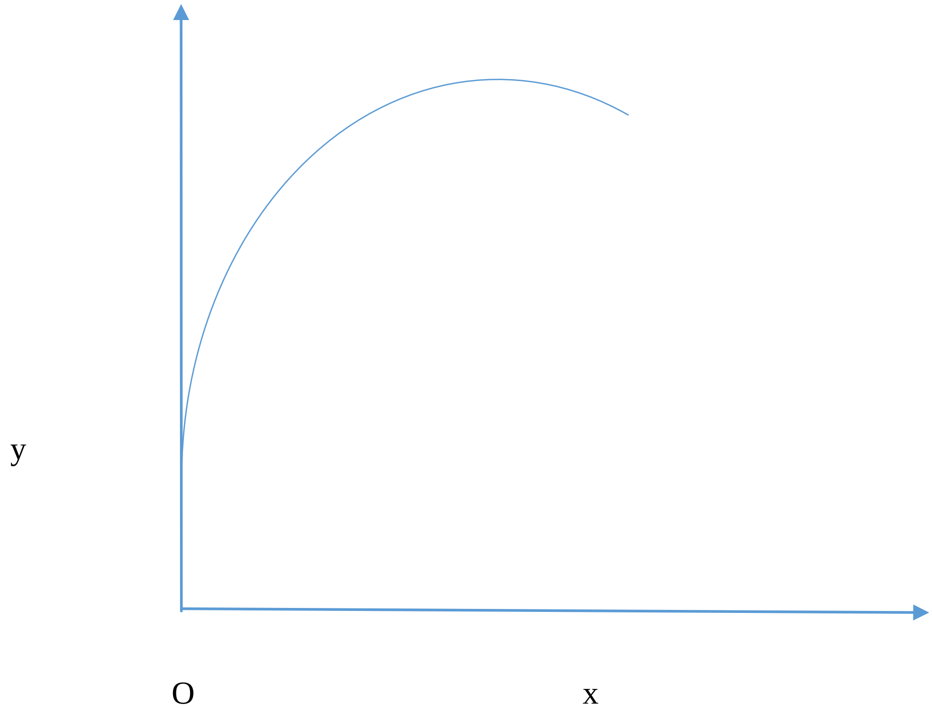


iii)x ά - y (x is directly proportional to negative of y)

x and y powers are same (equals to one). It means graph is a straight line



iv) if x ά y^2 power of x is one and power of y is 2 then graph is a parabola



Example:

• In the relation v= u + at for uniformly accelerated motion, the graph plotted between velocity and time a straight line as the degree of the equation is one.

Similarly in equation $S = ut+1/2\ at^2$ plotted graph must be a parabola. Since variable (S) has power of 1 and time (t) has power of 2….

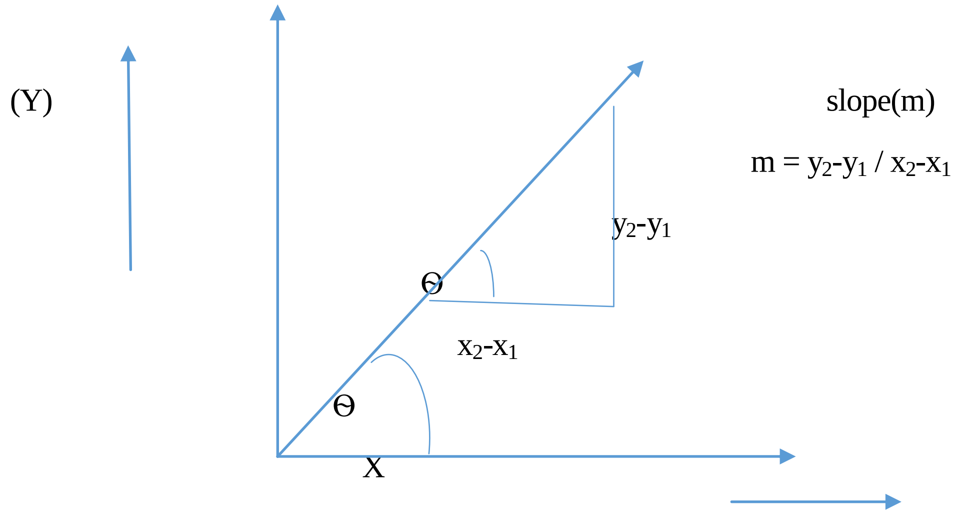
SLOPE OF A GIVEN CURVE (m)

• slope of a line or curve is a number , that gives both the direction and the steepness of the line. • It is denoted by the letter m •

Straight line equation $y = mx+c$

X and Y are variables and (m) is slope.

<u>TWO POINT METHOD OF CALCULATION OF SLOPE:</u>

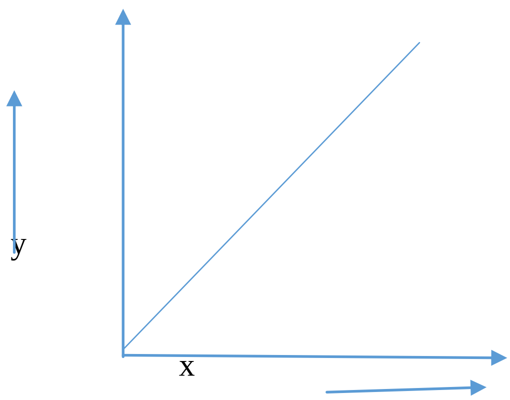


The slope m of a line is related to its angle of incline θ by the tangent

Slope (m) = $y_2 - y_1 / x_2 - x_1 = \tan\Theta$

<u>SIGN OF THE SLOPE :</u>

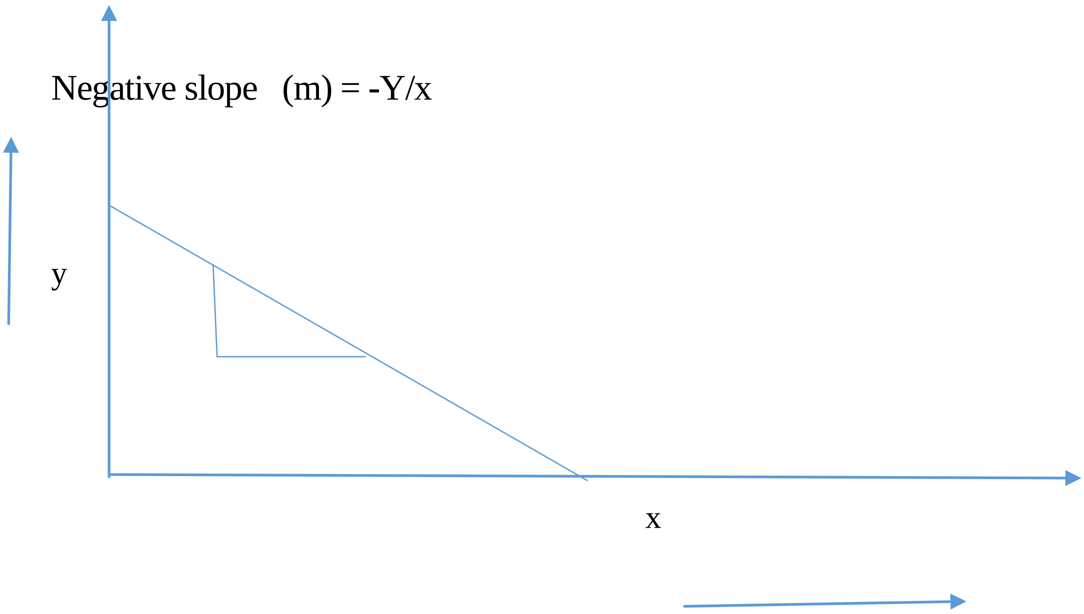
• A line goes up from left to right. The slope is positive. • If the angle between +x -axis and the line is acute, then the slope is positive. Below graph shows uniform motion with positive slope



NEGATIVE SLOPE • A line goes down from left to right. Then the slope of curve is negative.

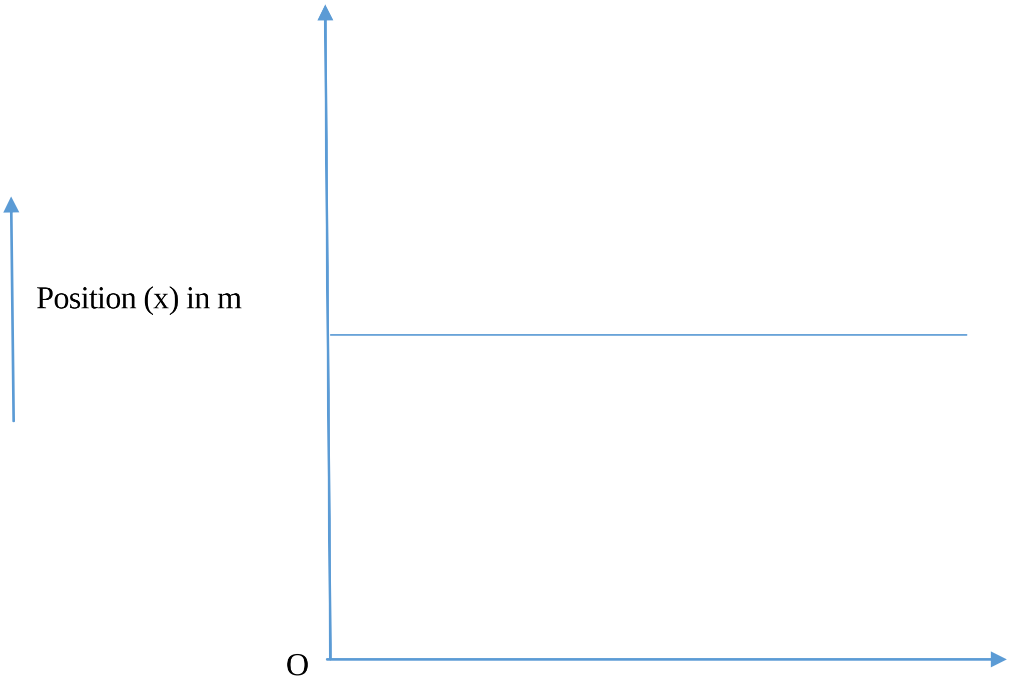
• The angle between + x- axis and the line or tangent to the curve is “obtuse.

Negative slope (m) = -Y/x



ZERO SLOPE

• If a line is horizontal (x-axis) the slope is zero



Time (t) in sec

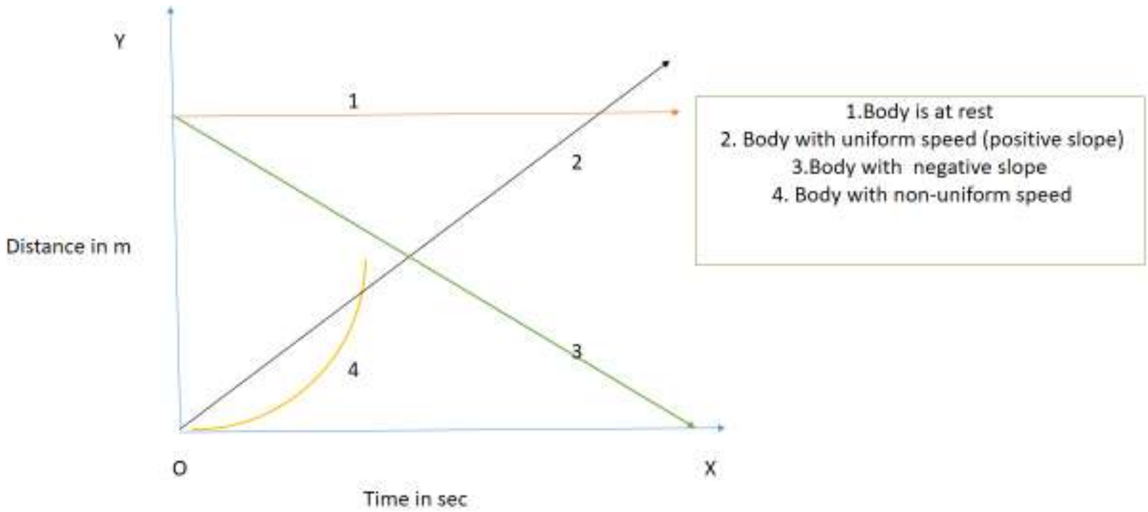


Figure: Distance- time graph of a body

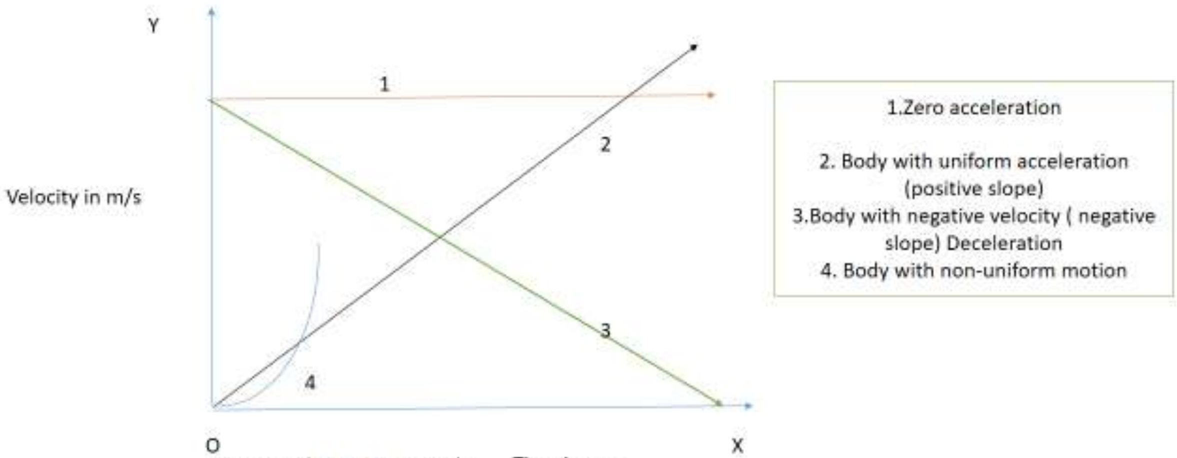


Figure: velocity- time graph

Derivation of kinematic equations for uniformly accelerated motion:

Equation for Velocity-Time relation by graphical method: (V = u + at)

Let an object is moving with uniform acceleration.

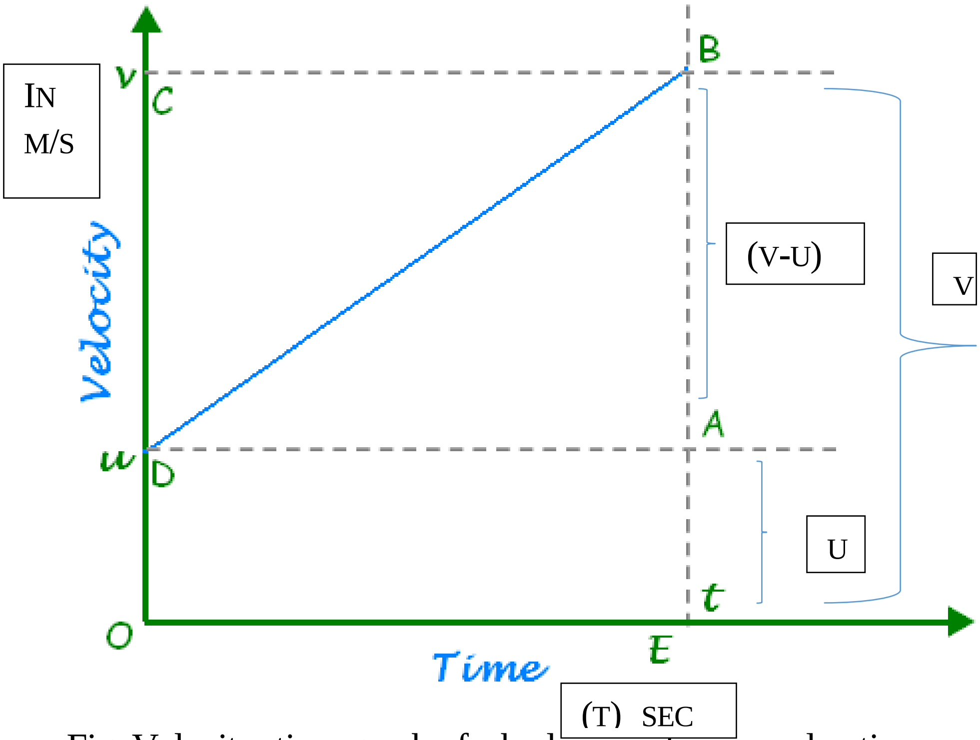


Fig. Velocity- time graph of a body in uniform acceleration:

Let body's

initial velocity = u , acceleration = a , final velocity becomes = v

Draw a line parallel to x-axis DA from point, D from where object starts moving.

Draw another line BA from point B parallel to y-axis which meets at Eat y-axis.

Let OE = time (t)

Now, from the graph,

BE = AB + AE

$\Rightarrow$ v = DC + OD (Since, AB = DC and AE = OD)

$\Rightarrow$ v = DC + u (Since, OD = u)

$\Rightarrow$ v = DC + u ------------------- (i)

We know that

Acceleration (a) = Change in velocity / Time taken

$\Rightarrow$a = (v−u)/t

$\Rightarrow$a = (v-u)/t

$\Rightarrow$a = (OC−OD)/ t = (DC)/t

$\Rightarrow$at=DC -----(ii)

By substituting the value of DC from (ii) in (i)

$\Rightarrow$v = u+at

Above equation is the relation among initial velocity (u), final velocity (v), acceleration (a) and time (t). It is called first equation of motion.

Equation for distance –time relation (s = ut +1/2at^2)

Distance covered by the object in the given time 't' is given by the area of the trapezium ABDOE

Let in the given time, (t) the distance covered by the moving object = (s)

The area of trapezium, ABDOE

Distance (s) = Area of △ABD+ Area of ADOE

$\Rightarrow$s=1/2×AB×AD+(OD×OE)

$\Rightarrow$s=12×AB×AD+(OD×OE)

$\Rightarrow$s=1/2×DC×AD+(ut)

$\Rightarrow$s=1/2×DC×AD+(ut)

[$\because$ AB=DC = at]

$\Rightarrow$s=1/2×at×t+ut

$\Rightarrow$s= ut +1/2at^2

Equation for Distance Velocity Relation: Third equation of Motion:

($v^2 - u^2 = 2as$)

The distance covered by the object moving with uniform acceleration is given by the area of trapezium ABDOE

area of trapezium

=1/2×(sum of parallel sides) Xdistance between parallel sides

⇒ Distance (s) =1/2(DO+BE)×OE

$\Rightarrow s=1/2(u+v)\times t$ ----(iii)

Now from equation (ii) $a= (v-u)/t$

$\therefore t = (v-u)/a$ ----(iv)

After substituting the value of t from equation (iv) in equation (iii)

$\Rightarrow s=1/2(u+v)\times(v-u)a$

$\Rightarrow s=1/2a(v+u)(v-u)$

$\Rightarrow 2as=(v+u)(v-u)$

$\Rightarrow 2as=v^2-u^2$

$\Rightarrow 2as+u^2=v^2$

$\Rightarrow v^2=u^2+2as$

(iv) Distance travelled in nth second.

$S_n = u + a / 2(2n - 1)$

If a body moves with uniform acceleration and velocity changes from u to v in a time interval, then the velocity at the mid point of its path

$\sqrt{u^2 + v^2 / 2}$

Equations of motion for constant acceleration using method of calculus:

1. Derivation of at = v-v0

By definition

a = dv/dt

integrate on both sides with in proper limits

$$\int adt = \int dv$$

$a\int_0^t dt = \int_{v0}^{v} dv$ v_0 = initial velocity , v= final velocity

a [t-0] = [v-v0]

at = v-v_0

v = v_0 +at

2) Derivation of s = ut +1/2at^2

V = dx/dt

d x = v dt

Integrating both sides$\int dx = \int vdt$

$$\int_{xo}^{x} dx = \int_{v0}^{v} (vo + at)dt$$

[x-x0] = $\int_0^t V0\; dt + \int_0^t at\; dt$

x-xo = v0$\int_0^t dt +$ a$\int_0^t tdt \int dx = x$, $\int tdt =$t2 /2

x-xo = v0 t +1/2 a t^2

x-xo = v0 t +1/2 a t^2

3. Derivation of v^2-u^2 = 2as

We know that

a =dv/dt

$=(dv/dx)\ (dx/dt)$

$a = V\ (dv/dx)$

$a\,dx= v\,dv$

$$\int_{x0}^{x} a dx = \int_{v0}^{v} v dv$$

$a[x- x_0] = 1/2\ [\ V^2\ ]^{v}_{vo}$

$a[x- x_0] =V^2 / 2 - V_o^2/2$

$V^2-V_0^2 = 2\ a[x- x_0]$

The advantage of this method is that it can be used for motion with non-uniform acceleration.

Kinematic equations for freely falling body:

A freely falling body always experience a downward acceleration g which we call as acceleration due to gravity.
Thus acceleration of freely falling body $a = g = 9.8\ m/s^2$ (Taking downward direction to be positive)
Initial speed of that body u=0
Equations of motion:

(i) We know that $V = u+at$
$V = -gt$

(ii) We know that $S = ut + at^2$

$S = -½\ gt^2$

(iii) We know that $v^2 - u^2 = 2as$

$V^2 = -2gS$

Galileo's law of odd numbers :

"The distances traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity [namely, 1: 3: 5: 7…...]."

For free fall
initial velocity (u) = 0 acceleration (a) = -g

let the distance (s) = y

we know that $S = ut + ½ at^2$

$y = -1/2gt^2$…………………….= y0

Using above equation we can calculate the position of the object after different intervals 0, τ, 2τ, 3τ……………. and $y0 = -1/2gt^2$

Table: for ratio of distances

T	Y	y in terms of y_0	Distances travelled in successive intervals	Ratio of distances
0	0	0		
T	$-1/2gt^2$	y_0	y_0	1
2τ	$-4 (1/2)gt^2$	$4 y_0$	$3 y_0$	3
3τ	$-9 (1/2)gt^2$	$9 y_0$	$5 y_0$	5
4 τ	$-16 (1/2)gt^2$	$16 y_0$	$7 y_0$	7
5 τ	$-25 (1/2)gt^2$	$25 y_0$	$9 y_0$	9
6 τ	$-36 (1/2)gt^2$	$36 y_0$	$11 y_0$	11

Stopping distance of vehicles :

When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance. It is an important factor for road safety and depends on the initial velocity (v0) and the braking capacity, or deceleration, (–a) that is caused by the braking.

Let the distance travelled by the vehicle before it stops be (ds)
Let initial velocity (u), Final velocity(v) = 0
We know that

$$V^2 - u^2 = 2as$$

$$-u^2 = 2a\, d_s$$

$$d_s = -u^2 / 2a$$

Thus, the stopping distance is proportional to the square of the initial velocity. Doubling theinitial velocity increases the stopping distance by a factor of 4 (for the same deceleration).

Reaction time :

When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appears on the road, then the time elapsed before he slams the brakes of the car is the reaction time. Reaction time depends on complexity of the situation and on an individual.

A stone is falling from top of building then personal on the ground to escape from accident must have a reaction time as follows.

For free fall (u) = 0 acceleration(a) = - height of building (d)

Let reaction time (tr)

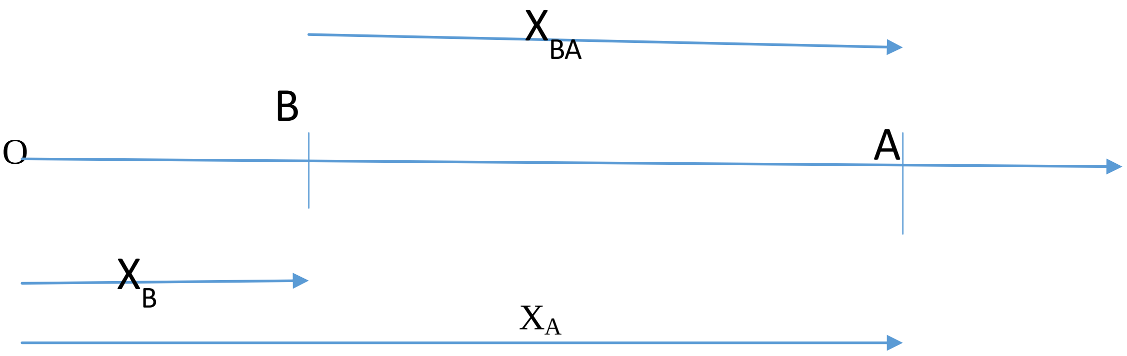
We know that $S = ut + ½ a t^2$

$d = -1/2\, g t_r^2$

$$t_r = \sqrt{2d/g}$$

<u>Relative velocity:</u> measurement of velocity of an object with respect to other object.

The time rate of change of relative position of one object with respect to other.



Let position of body A from origin o at time t = X_A

Let position of body B from origin o at time t = X_B

Relative position of B w.r.t to A is = X_{BA}

= $X_B - X_A$

Relative velocity of body B w.r.t body A is $V_{BA} = d/dt\,(X_B - X_A)$

$= V_B - V_A$

Sign convention: from above example

V_{BA}= positive means B is right of A (B along X-axis)

V_{BA} = negative means B is left of A (B along –X-axis)