

READING MATERIAL

LAWS OF MOTION

1. Force
2. Newton idea of force and laws of motion
3. Inertia (1) inertia of rest (2) inertia of motion (3) inertia of direction
4. Momentum
5. Newton's Laws of Motion
6. Impulse
7. Law of conservation of Momentum
8. Recoil of Gun
9. Weight, apparent weight, Atwood machine
10. Friction
11. Laws of friction
12. Angle of friction
13. Angle of repose
14. Methods to reduce friction
15. Free body diagram
16. Centripetal and centrifugal force.
17. Motion of a body on level road and on curved road
18. Banking of road
19. Equilibrium under concurrent forces.

LAWS OF MOTION

Dynamics

Dynamics is a branch of physics which deals with cause of motion.

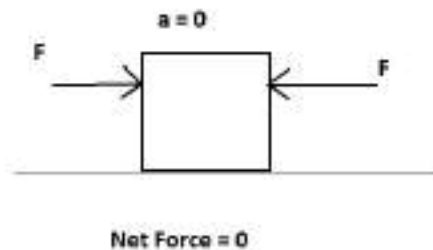
Force

Force is a quantity which tends to change the state of rest, state of motion, direction and shape of a body.

Types of forces

- 1) Balanced Force
- 2) Unbalanced force

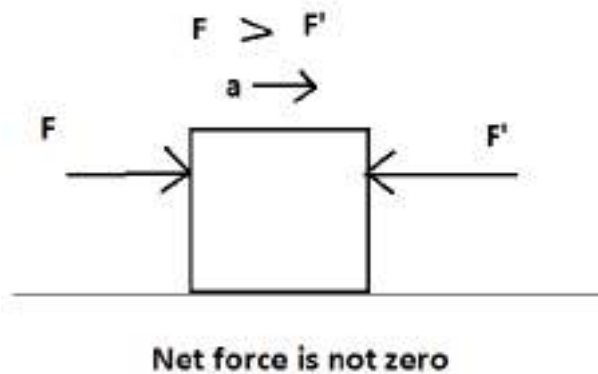
Balanced forces



Balanced forces are those forces which when act simultaneously on an object the resultant force is zero (acceleration of the body is zero)

Example: An object lying on the table, A man standing on ground, fan hanging from roof, a body moving with uniform velocity

Unbalanced forces



Unbalanced forces are those forces which when act simultaneously on an object the resultant force is non zero and produces acceleration in the body.

Example: A feely falling object under the influence of force of gravity, kicking a football, a train coming to state of rest after application of brake

Newton's First Law of Motion

A body continues to be in its state of rest or of uniform motion, unless it is acted upon by some external force.

- (1) If no net force acts on a body, then the velocity of the body cannot change *i.e.* the body cannot accelerate.
- (2) Newton's first law defines inertia and is rightly called the law of inertia.

Inertia

The property by virtue of which an object continues in the same state when an external force acts on it

Types of Inertia

- 1) Inertia of rest
- 2) Inertia of motion
- 3) Inertia of direction

Inertia of rest

It is the tendency of a body at rest to remain at rest.

Example:

A man standing in a stationary bus falls backward when the bus suddenly starts.

When a striker hits a pile of coins only the coins at the bottom of the pile move.

Inertia of Motion

It is the tendency of a body in uniform motion to remain at the same state.

Example:

A man standing in a moving bus falls forward when the bus suddenly stops.

An athlete warms up before participating in the event.

Inertia of Direction

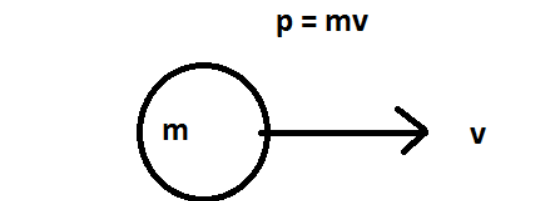
It is the tendency of a body moving in a direction to continue moving in the same direction.

Example:

Passengers sitting in a moving bus experience an outward force when the bus suddenly takes a turn.

Note: Force acting on a body changes its velocity depending upon its mass.

Momentum(\vec{p})



Momentum of an object is defined as the product of mass and velocity.

Momentum = mass x velocity

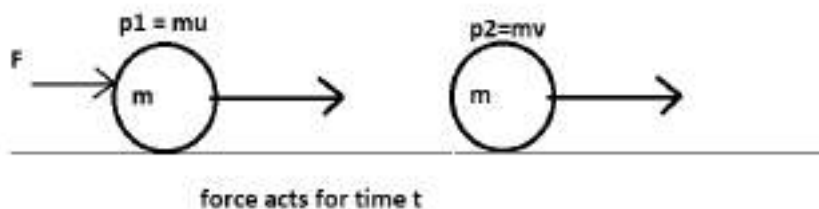
$$\vec{p} = m\vec{v}$$

It is a vector quantity.

Its unit is kgms^{-1}

Dimensional formula : $[\text{MLT}^{-1}]$

Newton's Second Law



(1) *The rate of change of linear momentum of a body is directly proportional to the external force applied on the body and this change takes place always in the direction of the applied force.*

(2) If a body of mass m , moves with velocity \vec{v} then its linear momentum can be given by

$$\vec{p} = m\vec{v}$$

and if force \vec{F} is applied on a body, then

$$\frac{d\vec{p}}{dt} = \vec{F}$$

($K = 1$ in C.G.S. and S.I. units)

$$\vec{F} = m\vec{a}$$

SI Unit of force is newton(N)

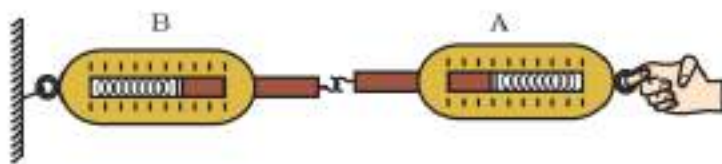
CGS unit of force is dyne

1N = 10^5 dyne

Definition of unit force.

One newton is that force which when acts on mass of 1kg produces an acceleration of 1ms^{-2} .

Newton's Third Law.



Action and reaction forces are equal and opposite.

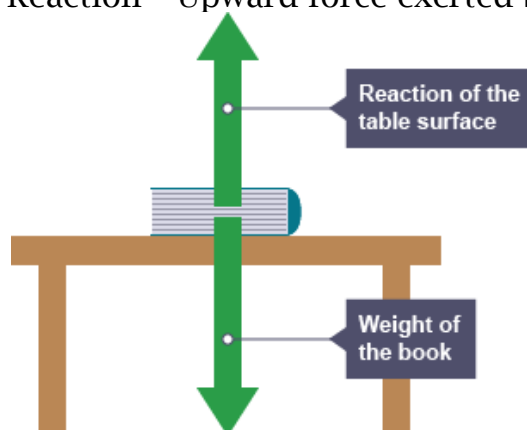
To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.

If F_{AB} = force exerted on body A by body B (Action) and F_{BA} = force exerted on body B by body A(Reaction).

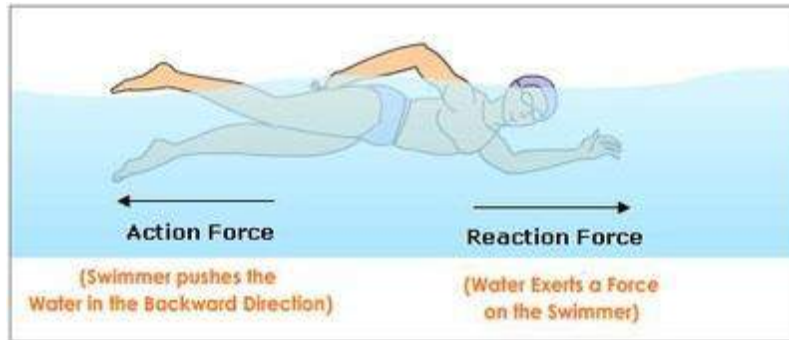
$$F_{AB} = -F_{BA}$$

Example:

- (i) A book lying on a table .
Action - Weight of book acting on the table.
Reaction - Upward force exerted by the table on the book.



- (ii) Swimming is possible due to third law of motion.
Action - Pushing the water backward.
Reaction - water pushes the swimmer in forward direction.



- (iii) When a gun is fired, the bullet moves forward (action). The gun recoils backward (reaction).

Newton's Second Law is the real law of motion

(i) Derivation first law from second law

According to second law

$$F = ma$$

If $F = 0$, then $a = 0$ (mass cannot be equal to zero)

If $a = 0$, velocity $v = \text{constant}$ or zero.

This means that it is at rest or moving with constant velocity.

This is what first law states.

(ii) Derivation third law from second law

$$F = ma$$

$$F_{21} = m_2 \left(\frac{v_2 - u_2}{t} \right)$$

$$F_{21}t = m_2v_2 - m_2u_2$$

$$F_{12} = m_1 \left(\frac{v_1 - u_1}{t} \right)$$

$$F_{12}t = m_1v_1 - m_1u_1$$

By law of conservation of momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1u_1 - m_1v_1 = m_2v_2 - m_2u_2$$

$$-F_{12}t = F_{21}t$$

$$-F_{12} = F_{21}$$

or

$$F_{21} = -F_{12}$$

Hence third law.

Since first and third law can be derived from second law therefore second law is the real law of motion.

Impulse

When a large force acts on a body for very small time interval, it is called impulsive force.

An impulsive force does not remain constant, but changes first from zero to maximum and then from maximum to zero. In such case we measure the total effect of force.

(2) Impulse of a force is a measure of total effect of force.

(3) Impulse is a vector quantity and its direction is same as that of force.

(4) Dimension: $[MLT^{-1}]$

(5) Units: *Newton-second* or $kg\cdot m\cdot s^{-1}$ (S.I.)



A fielder pulls his hands gradually with the moving ball while holding a catch.

Examples:

Hitting, kicking, catching, jumping, diving, collision *etc.*

In all these cases an impulse acts. . $I = \int F dt = F_{av} \cdot \Delta t = \Delta p = \text{constant}$

So if time of contact Δt is increased, average force is decreased and vice-versa.

(i) In catching a ball a player by drawing his hands backwards increases the time of contact and so, lesser force acts on his hands and his hands are saved from getting hurt.

(ii) China wares are wrapped in straw or paper before packing.

Law of Conservation of Linear Momentum

If no external force acts on a system, the total momentum of the system remains constant with time.

According to this law for a system of particles

$$\vec{F} = \frac{d\vec{p}}{dt}$$

In the absence of external force then,

$$\frac{d\vec{p}}{dt} = 0$$

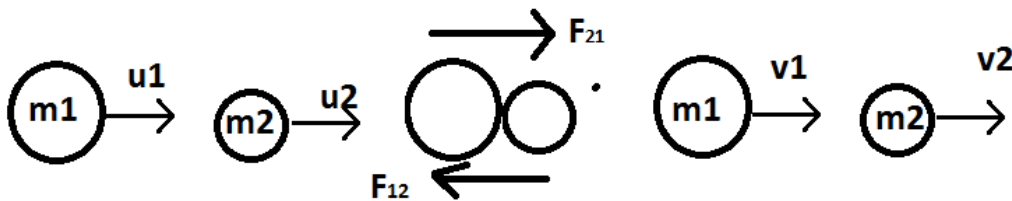
$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots = \text{constant}$$

Practical applications of the law of conservation of linear momentum

(i) When a man jumps out of a boat on the shore, the boat is pushed slightly away from the shore.

(ii) A person left on a frictionless surface can get away from it by blowing air out of his mouth or by throwing some object in a direction opposite to the direction in which he wants to move.

Proof



$$F_{21} = -F_{12}$$

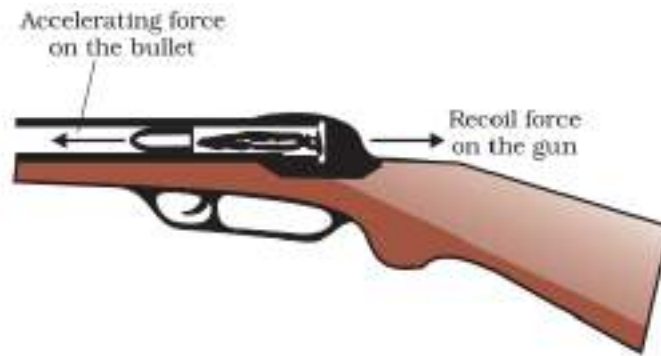
$$m_2 \left(\frac{v_2 - u_2}{t} \right) = -m_1 \left(\frac{v_1 - u_1}{t} \right)$$

$$m_2 v_2 - m_2 u_2 = -m_1 v_1 + m_1 u_1$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

Total momentum before collision = Total momentum after collision

Recoil of a gun



A forward force on the bullet and recoil of the gun.

The backward movement of gun after firing the bullet is called **recoil of gun**.

The velocity with which the gun moves backward is called **recoil velocity**.

Let M mass of gun, m mass of bullet, V recoil velocity of gun and v velocity of bullet.

momentum after firing = momentum before firing

$$MV = -mv$$

$$V = -\frac{mv}{M}$$

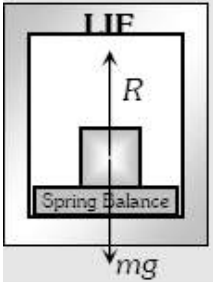
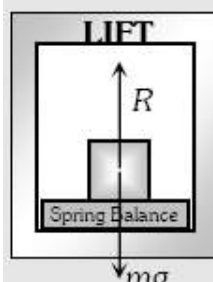
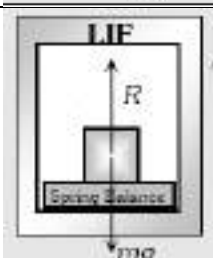
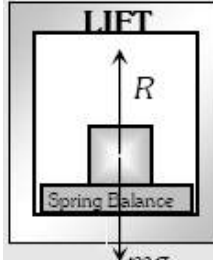
MASS AND WEIGHT

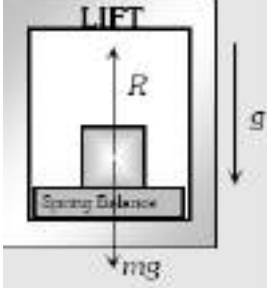
	MASS	WEIGHT
1	It is the amount of matter contained in a body.	It is the force with which the body is attracted to the centre of earth
2	$m = \frac{F}{a}$	$W = mg$
3	Unit :kilogram(kg)	Unit: newton(N)
4	It is scalar	It is a vector
5	It is measured using physical balance	It is measured using spring balance

Apparent Weight

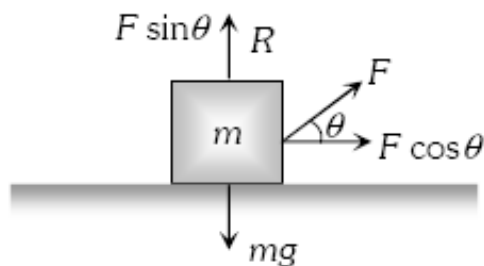
When a body of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the body is mg .

This acts on a weighing machine which offers a reaction R given by the reading of weighing machine. The reaction exerted by the surface of contact on the body is the *apparent weight* of the body.

Condition	Figure	Velocity	Acceleration	Reaction	Conclusion
Lift is at rest		$v = 0$	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift moving upward or downward with constant velocity		$v = \text{constant}$	$a = 0$	$R - mg = 0$ $\therefore R = mg$	Apparent weight = Actual weight
Lift accelerating upward at the rate of ' a '		$v = \text{variable}$	$a < g$	$R - mg = ma$ $\therefore R = m(g + a)$	Apparent weight > Actual weight
Lift accelerating downward at the rate of ' a '		$v = \text{variable}$	$a < g$	$mg - R = ma$ $\therefore R = m(g - a)$	Apparent weight < Actual weight

Lift accelerating downward at the rate of 'g'		$v =$ variable	$a = g$	$mg - R =$ mg $R = 0$	Apparent weight = Zero (weightlessness)
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Acceleration of Block on Horizontal Smooth Surface.



(1) When a pull is acting at an angle (θ) to the horizontal (upward)

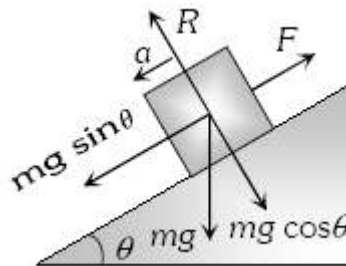
$$R + F \sin\theta = mg$$

$$R = mg - F \sin\theta$$

$$\text{and } F \cos\theta = ma$$

$$a = \frac{F \cos\theta}{m}$$

Acceleration of Block on Smooth Inclined Plane



(1) When inclined plane is at rest

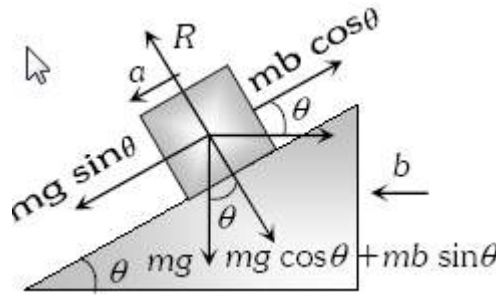
Normal reaction $R = mg \cos\theta$

Force along a inclined plane

$$F = mg \sin\theta$$

$$ma = mg \sin\theta$$

$$a = g \sin\theta$$



(2) When an inclined plane is given a horizontal acceleration ' b ' since the body lies in an accelerating frame, an inertial force (mb) acts on it in the opposite direction.

Normal reaction $R = mg \cos \theta + mb \sin \theta$

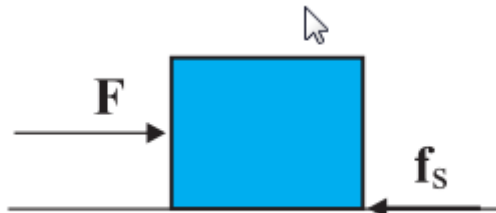
and $ma = mg \sin \theta - mb \cos \theta$

$a = g \sin \theta - b \cos \theta$

Friction

It is the opposing force which acts between any two surfaces in contact when there is a relative motion between them.

(1) Static friction:



The opposing force that comes into play when objects are at rest

(i) In this case static friction $f_s = F$

(ii) Static friction is a self-adjusting force because it changes itself in accordance with the applied force.

(2) Limiting friction:

It is the maximum value of force required to start the motion.

Laws of limiting friction

(i) The magnitude of limiting friction between any two surfaces in contact is directly proportional to the normal reaction between them.

$$f_s \propto R$$

$$f_s = \mu_s R$$

(iii) Direction of the force of limiting friction is always opposite to the direction in which one body is at the verge of moving

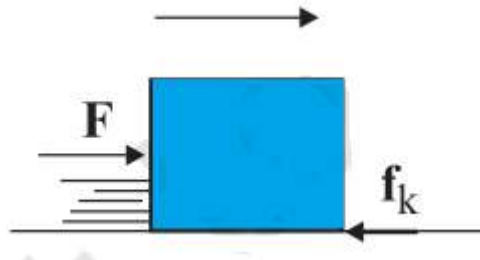
(iv) Limiting friction depends on material and nature of surfaces in contact.

- (v) Limiting friction does not depend on area of contact.

Coefficient of static friction

- (i) Dimension of μ_s : $[ML^0T^0]$
 (ii) Unit of μ_s : It has no unit.
 (iii) Value of μ_s lies in between 0 and 1

(3) Kinetic or dynamic friction



It is the friction which acts when the body is in motion.

- (i) Kinetic friction depends upon the normal reaction.

$F_k = \mu_k R$ or $F_k = \mu_k R$ where μ_k is called the coefficient of kinetic friction

- (ii) Kinetic friction is always lesser than limiting friction $F_k < F_s$ hence $\mu_k < \mu_s$

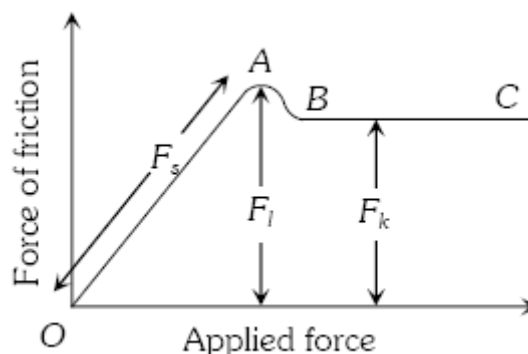
Thus we require more force to start a motion than to maintain it against friction this is to overcome inertia of rest. This is because when motion has actually started, irregularities of one surface have little time to get locked again into the irregularities of the other surface.

Types of kinetic friction: (a) Sliding friction (b) Rolling friction

Sliding friction is the friction which acts during sliding motion.

Rolling friction is the friction which acts during rolling motion.

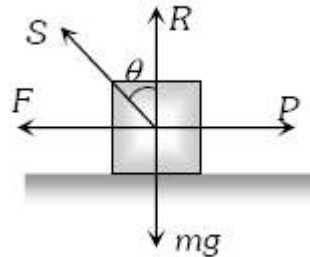
Graph between Applied Force and Force of Friction



- (1) Part OA = static friction (F_s).
 (2) At point A = limiting friction (F_l).
 (3) Beyond A , the force of friction is seen to decrease slightly. The portion BC = kinetic friction (F_k).

(4) As the portion BC of the curve is parallel to x -axis therefore kinetic friction does not change with the applied force.

Angle of Friction

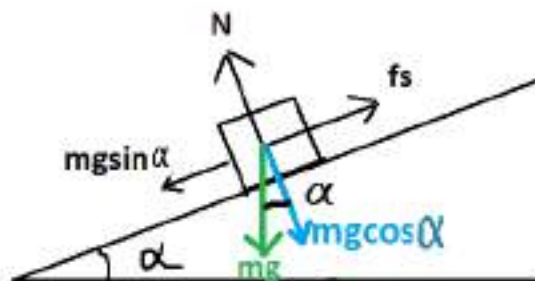


Angle of friction may be defined as the angle which the resultant of limiting friction and normal reaction makes with the normal reaction. By definition angle q is called the angle of friction

$$\tan \theta = \frac{F}{R}$$

$$\tan \theta = \mu$$

Angle of Repose



Angle of repose is defined as the angle of the inclined plane with horizontal such that a body placed on it is just begins to slide.

$$f_s = mg \sin \alpha$$

$$R = mg \cos \alpha$$

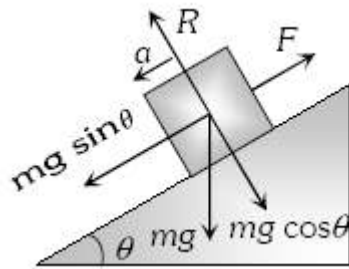
$$\frac{f_s}{R} = \tan \alpha$$

Therefore ,

$$\mu = \tan \alpha$$

If α is called the angle of repose $\alpha = \theta$ i.e. angle of repose = angle of friction.

Acceleration of a body down a rough inclined plane



When a plane is inclined to the horizontal at an angle θ , which is greater than angle of repose, the body placed on the inclined plane, slides down with an acceleration.

$$R = mg \cos\theta \quad (1)$$

net force on the Body down the inclined plane

$$f = mg \sin\theta - F \quad (2)$$

$$f = ma = mg \sin\theta - \mu R \quad (3)$$

$$ma = mg \sin\theta - \mu mg \cos\theta$$

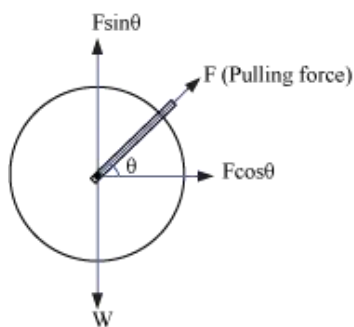
$$ma = mg(\sin\theta - \mu \cos\theta)$$

$$a = g(\sin\theta - \mu \cos\theta)$$

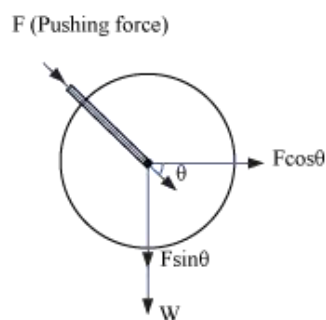
From above, we can understand the

$$a < g$$

Why it is easier to pull a lawn roller than to push



$$\text{Effective weight} = W - F \sin\theta$$



$$\text{Effective weight} = W + F \sin\theta$$

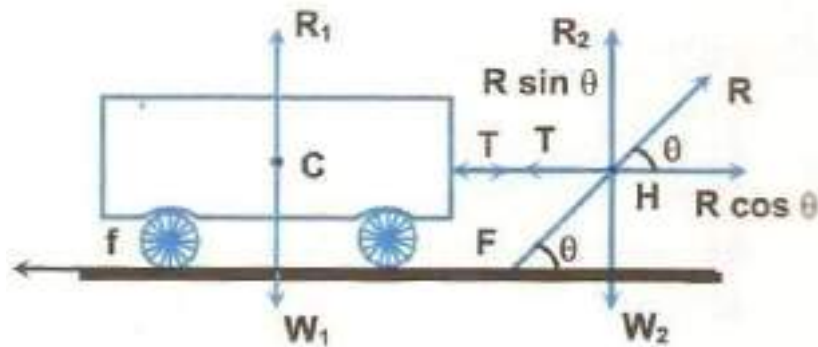
While pulling we have to apply force along the handle upwards. The vertical component of this force reduces the effective weight of the roller.

While in pushing force is applied along the handle downwards. The vertical component of this force increases the effective weight of the roller.

As the effective weight is lesser in pulling than in case of pushing, so pulling is easier than pushing

Horse and cart problem

When the horse pulls the cart, the cart pulls the horse with equal force in opposite direction. Then why does the cart move?



From the figure we find that $R_1 = W_1$ and $R_2 = W_2$

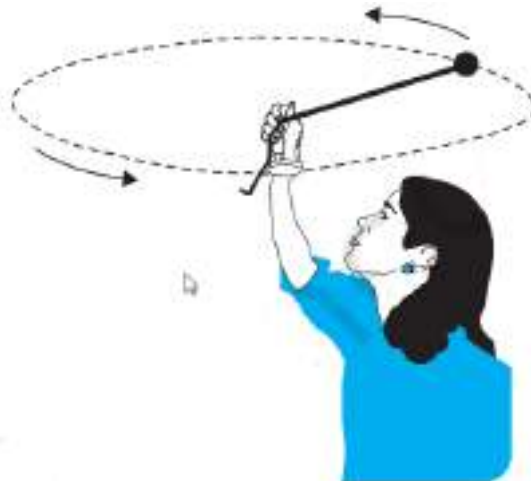
Tension in forward direction is equal to backward direction.

So horse and cart are in balanced state.

When horse pulls the cart, the horse pushes the ground with a force F in backward direction and reaction force R acts in opposite direction as shown in the figure. The components of R are also shown in the figure. The component $R \cos \theta$ acting in the forward direction pulls the cart.

If friction F acts between the cart and ground, then the cart will move only when $R \cos \theta > f$.

Uniform circular motion



Any body that moves in the circular path with a constant speed is said to be in uniform circular motion.

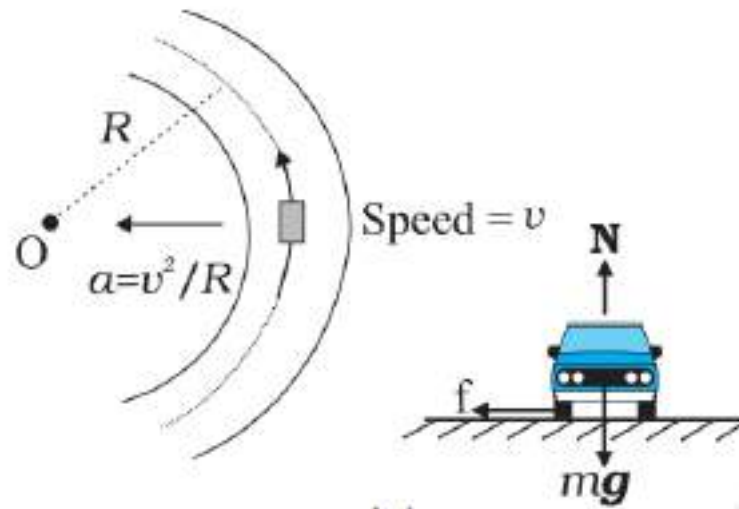
Centripetal force

The force required to keep the object in circular path is known as centripetal force. Centripetal force acts towards the centre of the circular path.

Consider an object of mass m moving with constant speed v along a circular path of radius r , experiences a centripetal force of

$$f = ma_c = \frac{mv^2}{R}$$

Motion of a vehicle on a level circular road



Three forces act on the car:

- (i) The weight of the car, mg
- (ii) Normal reaction, N
- (iii) Frictional force, f

As there is no acceleration in the vertical direction

$$N - mg = 0$$

$$N = mg$$

The centripetal force required for circular motion is along the surface of the road, and is provided by the frictional force between road and the car tyres along the surface.

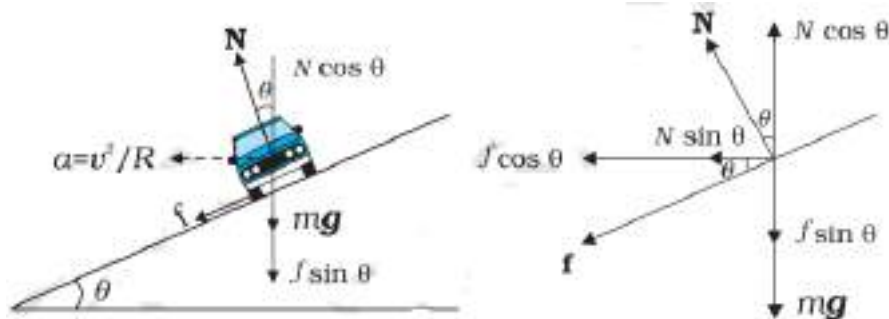
$$f = \frac{mv^2}{R} \leq \mu_s N$$
$$v^2 \leq \frac{\mu_s RN}{m} = \mu_s Rg$$

which is independent of the mass of the car.

This shows that for a given value of μ_s and R , there is a maximum speed of circular motion of the car possible, namely

$$v_{max} = \sqrt{\mu_s R g}$$

Motion of a car on a banked road



The raising of outer edge of a curved road above the inner edge is called banking of road.

We can reduce the contribution of friction to the circular motion of the car if the road is banked.

Since there is no acceleration along the vertical direction, the net force along this direction must be zero.

Hence

$$N \cos \theta = mg + f \sin \theta$$

The centripetal force is provided by the horizontal components of N and f

$$N \sin \theta + f \cos \theta = \frac{mv^2}{R}$$

$$f \leq \mu_s N$$

Thus to obtain v_{max} we put

$$f = \mu_s N$$

The above equation becomes

$$N \cos \theta = mg + \mu_s N \sin \theta$$

$$N \sin \theta + \mu_s N \cos \theta = \frac{mv^2}{R}$$

$$N = \frac{mg}{\cos \theta - \mu_s \sin \theta}$$

Substituting the value of N in above equation we get

$$\frac{mg(\sin \theta + \mu_s \cos \theta)}{(\cos \theta - \mu_s \sin \theta)} = \frac{mv_{max}^2}{R}$$

$$v_{max}^2 = Rg \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta}$$

$$v_{max} = \sqrt{Rg \frac{\mu_s + \tan \theta}{1 - \mu_s \tan \theta}}$$

Special cases

(i) If there is no friction then $\mu_s = 0$,

$$v_{max} = \sqrt{Rg \tan \theta}$$

(ii) If $\theta = 0$,

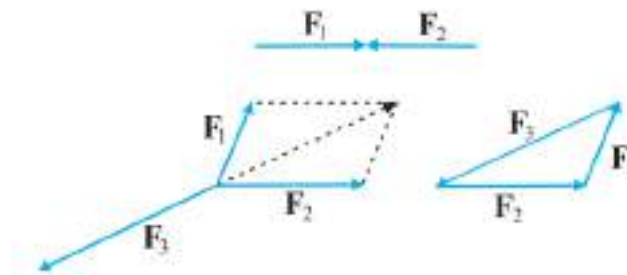
$$v_{max} = \sqrt{Rg\mu_s}$$

Bending of a cyclist

The angle by which a cyclist bends while negotiating a curved path path is given by

$$\tan \theta = \frac{v^2}{Rg}$$

Equilibrium under concurrent forces



A number of forces acting at a point is called concurrent forces.

Equilibrium of a particle in mechanics refers to the situation when the net external force on the particle is zero*. According to the first law, this means that, the particle is either at rest or in uniform motion.

If two forces F_1 and F_2 act on a particle, equilibrium requires

$$\mathbf{F}_1 = -\mathbf{F}_2$$

i.e. the two forces on the particle must be equal and opposite. Equilibrium under three concurrent forces F_1 , F_2 and F_3 requires that the vector sum of the three forces is zero.

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$$

In other words, the resultant of any two forces say F_1 and F_2 , obtained by the parallelogram law of forces must be equal and opposite to the third force, F_3

. As seen in Figure, the three forces in equilibrium can be represented by the sides of a triangle with the vector arrows taken in the same sense.

The result can be generalised to any number of forces. A particle is in equilibrium under the action of forces F_1, F_2, \dots, F_n if they can be represented by the sides of a closed n-sided polygon with arrows directed in the same sense.

Free Body Diagram

In this diagram the object of interest is isolated from its surroundings and the interactions between the object and the surroundings are represented in terms of forces.

Motion of blocks in contact

		$F - f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$
Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \frac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - f = m_2 a$	$f = \frac{m_2 F}{m_1 + m_2}$

		$f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_2 - f_1 = m_2 a$	
		$F - f_2 = m_3 a$	

Motion of blocks connected by mass less string

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - T = m_2 a$	
		$F - T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$T = m_2 a$	

		$T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	
		$F - T_2 = m_3 a$	
		$F - T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_1 - T_2 = m_2 a$	
		$T_2 = m_3 a$	

Motion of Connected Block over a Pulley

Condition	Free body diagram	Equation	Tension and acceleration
When pulley have a finite mass M and radius R then tension in two segments of string are different		$m_1 a = m_1 g - T_1$	$a = \frac{m_1 - m_2}{m_1 + m_2 + \frac{M}{2}}$ $T_1 = \frac{m_1 \left[2m_2 + \frac{M}{2} \right]}{m_1 + m_2 + \frac{M}{2}} g$ $T_2 = \frac{m_2 \left[2m_1 + \frac{M}{2} \right]}{m_1 + m_2 + \frac{M}{2}} g$
		$m_2 a = T_2 - m_2 g$	
		Torque $-(T_1 - T_2)R = I\alpha$ $(T_1 - T_2)R = I \frac{a}{R}$ $(T_1 - T_2)R = \frac{1}{2} MR^2 \frac{a}{R}$ $T_1 - T_2 = \frac{M a}{2}$	

		$m_1 a = T_1 - m_1 g$	$T_1 = \frac{2m_1(m_2 + m_3)}{m_1 + m_2 + m_3} g$
		$m_2 a = m_2 g + T_1 - T_2$	$T_2 = \frac{2m_2 m_3}{m_2 + m_2 + m_3} g$
		$m_3 a = m_3 g - T_2$	$T_3 = \frac{4m_3(m_2 + m_3)}{m_1 + m_2 + m_3} g$
		$T_3 = 2T_1$	$a = \frac{[m_2 + m_3 - m_1]g}{m_1 + m_2 + m_3}$

		$T = m_1 a$	$a = \frac{m_2}{m_1 + m_2} g$
		$m_2 a = m_2 g - T$	$T = \frac{m_1 m_2}{m_1 + m_2} g$