STUDY MATERIALS:-

Complex numbers are the numbers which are expressed in the form of **a+ib** where i is an imaginary number called ***iota*** and has the value of (√-1). For example, 2+3i is a complex number, where 2 is a real number and 3i is an imaginary number.

Therefore, the **combination of both the real number and imaginary number is a complex number**.

The main application of these numbers is to represent periodic motions such as water waves, alternating current, light waves, etc., which relies on sine or cosine waves etc. There are certain formulas which are used to solve the problems based on complex numbers. Also, the mathematical operations such as addition, subtraction and multiplication are performed on these numbers. The key concepts are highlighted in this article will include the following:

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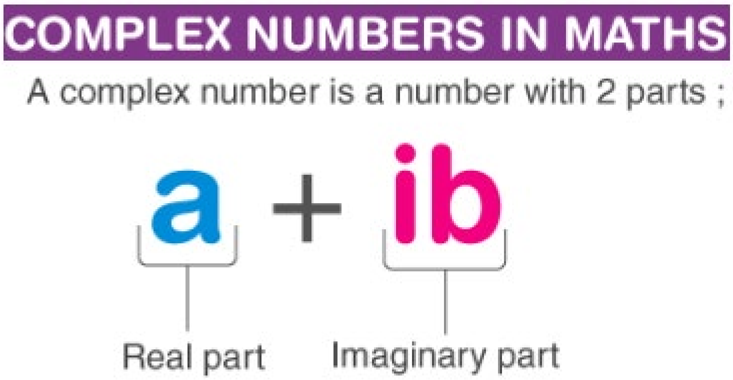
What are Complex Numbers?

The complex number is basically the combination of a real number and an imaginary number. The complex number is of the form **a+ib**. The real numbers are the numbers which we usually work on to do the mathematical calculations. But the imaginary numbers are not generally used for calculations but only in the case of imaginary numbers. let us check the definitions for both the numbers.



What are Real Numbers?

Any number which is present in a number system such as positive, negative, zero, integer, rational, irrational, fractions, etc. are real numbers. It is represented as Re(part). For example: 12, -45, 0, 1/7, 2.8, √5 are all real numbers.

What are Imaginary Numbers?

The numbers which are not real are imaginary numbers. When we square an imaginary number, it gives a negative result. It is represented as Im(). Example: √-2, √-7, √-11 are all imaginary numbers.

In the 16th century, the complex numbers were introduced which made it possible to solve the equation $x^2 + 1 = 0$. The roots of the equation are of form $x = \pm\sqrt{-1}$ and no real roots exist. Thus, with the introduction of complex numbers, we have Imaginary roots.

We denote √-1 with the symbol 'i', where i denotes Iota (Imaginary number).

An equation of the form z= a+ib, where a and b are real numbers, is defined to be a complex number. The real part is denoted by Re z = a and the imaginary part is denoted by Im z = ib.

See the table below to differentiate between a real number and an imaginary number.

Complex Number	**Real Number**	**Imaginary Number**
-1+2i	-1	2i
7-9i	7	-9i
-6i	0	-6i(Purely Imaginary)
6	6	0i(Purely Real)

Algebraic Operations on Complex Numbers:-

There can be four types of algebraic operation on complex numbers which are mentioned below. Visit the linked article to know more about these algebraic operations along with solved examples. The four operations on the complex numbers include:

- Addition
- Subtraction
- Multiplication
- Division

Roots of Complex Numbers

When we solve a quadratic equation of the form $ax^2 + bx + c = 0$, the roots of the equations can be determined in three forms;

- Two Distinct Real Roots
- Similar Root
- No Real roots (Complex Roots)

Complex Number Formulas:-

Addition

$(a + ib) + (c + id) = (a + c) + i(b + d)$

Subtraction

$(a + ib) – (c + id) = (a – c) + i(b – d)$

Multiplication

$(a + ib). (c + id) = (ac – bd) + i(ad + bc)$

Division

$(a + ib) / (c + id) = (ac+bd)/ (c^2 + d^2) + i(bc – ad) / (c^2 + d^2)$

- Complex Numbers and Quadratic Equations
- Complex Number Formula
- Complex Numbers Class 11

Power of Iota (i)

Depending upon the power of "i", it can take the following values;

$i^{4k+1} = i$. $i^{4k+2} = -1$ $i^{4k+3} = -i$. $i^{4k} = 1$

where k can have an integral value (positive or negative).

Similarly, we can find for the negative power of *i*, which are as follows;

$i^{-1} = 1 / i$

Multiplying and dividing the above term with i, we have;

$i^{-1} = 1 / i \times i/i \times i^{-1} = i / i^2 = i / -1 = -i / -1 = -i$

Note:

$\sqrt{-1} \times \sqrt{-1} = \sqrt{(-1 \times -1)} = \sqrt{1} = 1$ contradicts to the fact that $i^2 = -1$.

Therefore, for an imaginary number, $\sqrt{a} \times \sqrt{b}$ is not equal to $\sqrt{ab}$.

Complex Numbers Identities

Let us see some of the identities.

1. $(z_1 + z_2)^2 = (z_1)^2 + (z_2)^2 + 2\, z_1 \times z_2$
2. $(z_1 - z_2)^2 = (z_1)^2 + (z_2)^2 - 2\, z_1 \times z_2$
3. $(z_1)^2 - (z_2)^2 = (z_1 + z_2)(z_1 - z_2)$
4. $(z_1 + z_2)^3 = (z_1)^3 + 3(z_1)^2 z_2 + 3(z_2)^2 z_1 + (z_2)^3$
5. $(z_1 - z_2)^3 = (z_1)^3 - 3(z_1)^2 z_2 + 3(z_2)^2 z_1 - (z_2)^3$

Modulus and Conjugate

Let z=a+ib be a complex number.

The **Modulus of z** is represented by |z|.

Mathematically, |z|=a2+b2−−−−−−√

The **conjugate of "z"** is denoted by z¯.

Mathematically, z¯ = a – ib

Argand Plane and Polar Representation:-

Similar to the XY plane, the Argand(or complex) plane is a system of rectangular coordinates in which the complex number a+ib is represented by the point whose coordinates are a and b.

We find the real and complex components in terms of r and θ, where r is the length of the vector and θ is the angle made with the real axis. Check out the detailed argand plane and polar representation of complex numbers in this article and understand this concept in a detailed way along with solved examples.

Complex Numbers Problems

Example 1:

Simplify

a) 16i + 10i(3-i)

b) (7i)(5i)

c) 11i + 13i – 2i

Solution:

a) 16i + 10i(3-i)

= 16i + 10i(3) + 10i (-i)

= 16i +30i – 10 i^2

= 46 i – 10 (-1)

= 46i + 10

b) (7i)(5i) = 35 i^2 = 35 (-1) = -35

c) 11i + 13i – 2i = 22i

Example 2:

Express the following in a+ib form

(5+√3i)/(1-√3i).

Solution:

Given: (5+√3i)/(1-√3i)

$$z = \frac{5+i\sqrt{3}}{1-i\sqrt{3}} \times \frac{1+i\sqrt{3}}{1+i\sqrt{3}} = \frac{5-3+6\sqrt{3}i}{1+3} = \frac{1}{2} + \frac{3\sqrt{3}}{2}i$$

$$\text{Modulus, } \bar{z} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} = \sqrt{\frac{28}{4}} = \sqrt{7}$$

$$\text{Conjugate, } \bar{z} = \frac{1}{2} - \frac{3\sqrt{3}}{2}i$$

Learn more about the Identities, Conjugate of the complex number, and other complex numbers related concepts at BYJU'S. Also, get additional study materials for various maths topics along with practice questions, examples, and tips to be able to learn maths more effectively.

JEE Main Maths Complex Numbers & Quadratic Equations Previous Year Questions With Solutions

Question 1: If (1 + i) (1 + 2i) (1 + 3i) ….. (1 + ni) = a + ib, then what is 2 * 5 * 10….$(1 + n^2)$ is equal to?

Solution:

We have (1 + i) (1 + 2i) (1 + 3i) ….. (1 + ni) = a + ib …..(i)

(1 − i) (1 − 2i) (1 − 3i) ….. (1 − ni) = a − ib …..(ii)

Multiplying (i) and (ii),

we get 2 * 5 * 10 ….. $(1 + n^2) = a^2 + b^2$

Question 2: If z is a complex number, then the minimum value of $|z| + |z - 1|$ is ______.

Solution:

First, note that $|-z|=|z|$ and $|z_1 + z_2| \le |z_1| + |z_2|$

Now $|z| + |z - 1| = |z| + |1 - z| \ge |z + (1 - z)|$

$= |1| = 1$

Hence, minimum value of $|z| + |z - 1|$ is 1.

Question 3: For any two complex numbers z1 and z2 and any real numbers a and b; $|(az_1 - bz_2)|^2 + |(bz_1 + az_2)|^2$ = ___________.

Solution:

$|(az_1 - bz_2)|^2 + |(bz_1 + az_2)|^2$

$= a_2 |z_1|^2 + b_2|z_2|^2 - 2 \text{ Re } (ab)|z_1$\overline{z2}$z2| + b2 |z_1|^2 + a_2|z_2|^2 + 2 \text{ Re } (ab)|$\overline{z1}$z1z2|$

$= (a_2 + b_2) (|z_1|^2 + |z_2|^2)$

Question 4: Find the complex number z satisfying the equations |\frac{z-12}{z-8i}| = \frac{5}{3}$|z-8iz-12|=35$, |\frac{z-4}{z-8}| = 1$|z-8z-4|=1$

Solution:

We have |\frac{z-12}{z-8i}| = \frac{5}{3}$|z-8iz-12|=35$, |\frac{z-4}{z-8}| = 1$|z-8z-4|=1$

Let z = x + iy, then |\frac{z-12}{z-8i}| = \frac{5}{3}$|z-8iz-12|=35$

$\Rightarrow 3|z - 12| = 5 |z - 8i|$

$3\,|(x - 12) + iy| = 5\,|x + (y - 8)\,i|$

$9\,(x - 12)^2 + 9y^2 = 25x^2 + 25\,(y - 8)^2$ ….(i) and

$\left|\frac{z-4}{z-8}\right| = 1$

$\Rightarrow |z - 4| = |z - 8|$

$|x - 4 + iy| = |x - 8 + iy|$

$(x - 4)^2 + y^2 = (x - 8)^2 + y^2$

$\Rightarrow x = 6$

Putting $x = 6$ in (i), we get $y^2 - 25y + 136 = 0$

$y = 17, 8$

Hence, $z = 6 + 17i$ or $z = 6 + 8i$

Question 5: If $z_1 = 10 + 6i$, $z_2 = 4 + 6i$ and z is a complex number such that amp $\frac{z-z_1}{z-z_2} = \frac{\pi}{4}$, then the value of $|z - 7 - 9i|$ is equal to _________.

Solution:

Given numbers are $z_1 = 10 + 6i$, $z_2 = 4 + 6i$ and $z = x + iy$

amp $\frac{z-z_1}{z-z_2} = \frac{\pi}{4}$

amp $[(x - 10) + i\,(y - 6)\,(x - 4) + i\,(y - 6)] = \pi / 4$

$\frac{(x - 4)(y - 6) - (y - 6)(x - 10)}{(x - 4)(x - 10) + (y - 6)^2} = 1$

$12y - y^2 - 72 + 6y = x^2 - 14x + 40$ …..(i)

Now $|z - 7 - 9i| = |(x - 7) + i\,(y - 9)|$

From (i), $(x^2 - 14x + 49) + (y^2 - 18y + 81) = 18$

$(x - 7)^2 + (y - 9)^2 = 18$ or

$[(x - 7)^2 + (y - 9)^2]^{½} = [18]^{½} = 3\sqrt{2}$

$|(x - 7) + i\,(y - 9)| = 3\sqrt{2}$ or

$|z - 7 - 9i| = 3\sqrt{2}$.

Question 6: Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find the values of z_3 and z_2.

Solution:

One of the numbers must be a conjugate of $z_1 = 1 + i\sqrt{3}$ i.e. $z_2 = 1 - i\sqrt{3}$ or $z_3 = z_1 e^{i2\pi/3}$ and

$z_2 = z_1 e^{-i2\pi/3}$, $z_3 = (1 + i\sqrt{3})\,[\cos(2\pi / 3) + i \sin(2\pi / 3)] = -2$.

Question 7: If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then what is the value of $\cos 3\alpha + \cos 3\beta + \cos 3\gamma$?

Solution:

$\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$

Let $a = \cos\alpha + i\sin\alpha$; $b = \cos\beta + i\sin\beta$ and $c = \cos\gamma + i\sin\gamma$.

Therefore, $a + b + c = (\cos\alpha + \cos\beta + \cos\gamma) + i(\sin\alpha + \sin\beta + \sin\gamma) = 0 + i0 = 0$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$ or

$(\cos\alpha + i\sin a)^3 + (\cos\beta + i\sin\beta)^3 + (\cos\gamma + i\sin\gamma)^3$

$= 3(\cos\alpha + i\sin\alpha)(\cos\beta + i\sin\beta)(\cos\gamma + i\sin\gamma)$

$\Rightarrow (\cos3\alpha + i\sin3\alpha) + (\cos3\beta + i\sin3\beta) + (\cos3\gamma + i\sin3\gamma)$

$= 3[\cos(\alpha + \beta + \gamma) + i\sin(\alpha + \beta + \gamma)]$ or $\cos 3\alpha + \cos 3\beta + \cos 3\gamma$

$= 3\cos(\alpha + \beta + \gamma)$.

Question 8: If the cube roots of unity are $1, \omega, \omega^2$, then find the roots of the equation $(x - 1)^3 + 8 = 0$.

Solution:

$(x - 1)^3 = -8 \Rightarrow x - 1 = (-8)^{1/3}$

$x - 1 = -2, -2\omega, -2\omega^2$

$x = -1, 1 - 2\omega, 1 - 2\omega^2$

Question 9: If $1, \omega, \omega^2, \omega^3 \ldots\ldots., \omega^{n-1}$ are the n, n^{th} roots of unity, then $(1 - \omega)(1 - \omega^2) \ldots..$

$(1 - \omega^{n-1})$ = ____________.

Solution:

Since 1, ω, ω^2, ω^3……., ω^{n-1} are the n, n^{th} roots of unity, therefore, we have the identity

$= (x - 1)(x - \omega)(x - \omega^2) \ldots (x - \omega^{n-1}) = x^n - 1$ or

$(x - \omega)(x - \omega^2) \ldots (x - \omega^{n-1}) = x^n - 1 / x - 1$

$= x^{n-1} + x^{n-2} + \ldots + x + 1$

Putting x = 1 on both sides, we get

$(1 - \omega)(1 - \omega^2) \ldots (1 - \omega^{n-1}) = n$

Question 10: If a = cos (2π / 7) + i sin (2π / 7), then the quadratic equation whose roots are $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$ is _____________.

Solution:

a = cos (2π / 7) + i sin (2π / 7)

$a^7 = [\cos(2\pi / 7) + i \sin(2\pi / 7)]^7$

= cos 2π + i sin 2π = 1 …..(i)

$S = \alpha + \beta = (a + a^2 + a^4) + (a^3 + a^5 + a^6)$

$S = a + a^2 + a^3 + a^4 + a^5 + a^6$

$= \frac{a(1-a^6)}{1-a}$

or $S = \frac{a-1}{1-a}$

= −1 …..(ii)

$P = \alpha * \beta = (a + a^2 + a^4)(a^3 + a^5 + a^6)$

$= a^4 + a^6 + a^7 + a^5 + a^7 + a^8 + a^7 + a^9 + a^{10}$

$= a^4 + a^6 + 1 + a^5 + 1 + a + 1 + a^2 + a^3$ (From eqn (i)]

$= 3 + (a + a^2 + a^3 + a^4 + a^5 + a^6)$

= 3 + S = 3 − 1 = 2 [From (ii)]

Required equation is, $x^2 - Sx + P = 0$

$x^2 + x + 2 = 0$.

Question 11: Let z_1 and z2 be nth roots of unity, which are ends of a line segment that subtend a right angle at the origin. Then n must be of the form ____________.

Solution:

$1^{1/n} = \cos [2r\pi / n] + i \sin [2r\ \pi / n]$

Let $z_1 = [\cos 2r_1\pi / n] + i \sin [2r_1\pi / n]$ and $z_2 = [\cos 2r_2\pi / n] + i \sin [2r_2\pi / n]$.

Then $\angle Z_1 O Z_2 = \text{amp}\ (z_1 / z_2) = \text{amp}\ (z_1) - \text{amp}\ (z_2)$

$= [2\ (r_1 - r_2)\pi] / [n]$

$= \pi / 2$

(Given) $n = 4\ (r_1 - r_2)$

= 4 × integer, so n is of the form 4k.

Question 12: $(\cos \theta + i \sin \theta)^4 / (\sin \theta + i \cos \theta)^5$ is equal to ____________.

Solution:

$(\cos \theta + i \sin \theta)^4 / (\sin \theta + i \cos \theta)^5$

$= (\cos \theta + i \sin \theta)^4 / i^5 ([1 / i] \sin \theta + \cos \theta)^5$

$= (\cos\theta + i \sin \theta)^4 / i (\cos \theta - i \sin \theta)^5$

$= (\cos \theta + i \sin \theta)^4 / i (\cos \theta + i \sin \theta)^{-5}$ (By property) $= 1 / i (\cos \theta + i \sin \theta)^9$

$= \sin(9\theta) - i \cos (9\theta)$.

Question 13: Given $z = (1 + i\sqrt{3})^{100}$, then find the value of Re (z) / Im (z).

Solution:

Let $z = (1 + i\sqrt{3})$

$r = \sqrt{[3 + 1]} = 2$ and $r \cos\theta = 1$, $r \sin\theta = \sqrt{3}$ $\tan\theta = \sqrt{3} = \tan \pi / 3 \Rightarrow \theta = \pi / 3$.

$z = 2 (\cos \pi / 3 + i \sin \pi / 3)$

$z^{100} = [2 (\cos \pi / 3 + i \sin \pi / 3)]^{100}$

$= 2^{100} (\cos 100\pi / 3 + i \sin 100\pi / 3)$

$= 2^{100} (-\cos \pi / 3 - i \sin \pi / 3)$

$= 2^{100}(-1 / 2 - i \sqrt{3} / 2)$

$Re(z) / Im(z) = [-1/2] / [-\sqrt{3} / 2] = 1 / \sqrt{3}$.

Question 14: If $x = a + b$, $y = a\alpha + b\beta$ and $z = a\beta + b\alpha$, where α and β are complex cube roots of unity, then what is the value of xyz?

Solution:

If $x = a + b$, $y = a\alpha + b\beta$ and $z = a\beta + b\alpha$, then $xyz = (a + b) (a\omega + b\omega^2) (a\omega^2 + b\omega)$,where $\alpha = \omega$ and $\beta = \omega^2 = (a + b) (a^2 + ab\omega^2 + ab\omega + b^2)$

$= (a + b) (a^2 - ab + b^2)$

$= a^3 + b^3$

Question 15: If ω is an imaginary cube root of unity, $(1 + \omega - \omega^2)^7$ equals to ___________.

Solution:

$(1 + \omega - \omega^2)^7 = (1 + \omega + \omega^2 - 2\omega^2)^7$

$= (-2\omega^2)^7$

$= -128\omega^{14}$

$= -128\omega^{12}\omega^2$

$= -128\omega^2$

Question 16: If α, β, γ are the cube roots of p ($p < 0$), then for any x, y and z, find the value of $[x\alpha + y\beta + z\gamma] / [x\beta + y\gamma + z\alpha]$.

Solution:

Since $p < 0$.

Let $p = -q$, where q is positive.

Therefore, $p^{1/3} = -q1^{/3}(1)^{1/3}$.

Hence $\alpha = -q^{1/3}$, $\beta = -q^{1/3}\omega$ and $\gamma = -q^{1/3}\omega^2$

The given expression

$[x + y\omega + z\omega^2] / [x\omega + y\omega^2 + z] = (1 / \omega) * [z\omega + y\omega^2 + z] / [x\omega + y\omega^2 + z]$

$= \omega^2$.

Question 17:

The common roots of the equations $x^{12} - 1 = 0$, $x^4 + x^2 + 1 = 0$ are __________.

Solution:

$x^{12} - 1 = (x^6 + 1)(x^6 - 1)$

$= (x^6 + 1)(x^2 - 1)(x^4 + x^2 + 1)$

Common roots are given by $x^4 + x^2 + 1 = 0$

$x^2 = [-1 \pm i\sqrt{3}] / [2] = \omega, \omega^2$ or ω^4, ω^2 (Because $\omega^3 = 1$) or

$x = \pm\omega^2, \pm\omega$.

Question 18: Given that the equation $z^2 + (p + iq)z + r + is = 0$, where p, q, r, s are real and non-zero has a real root, then how are p, q, r and s related?

Solution:

Given that $z^2 + (p + iq)z + r + is = 0$ ……(i)

Let $z = \alpha$ (where α is real) be a root of (i), then

$\alpha^2 + (p + iq)\alpha + r + is = 0$ or

$\alpha^2 + p\alpha + r + i(q\alpha + s) = 0$

Equating real & imaginary parts, we have $\alpha^2 + p\alpha + r = 0$ and $q\alpha + s = 0$

Eliminating α, we get

$(-s / q)^2 + p(-s / q) + r = 0$ or

$s^2 - pqs + q^2r = 0$ or

$pqs = s^2 + q^2r$

Question 19: The difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then what is the relation between a and b?

Solution:

Let α, β and γ,δ be the roots of the equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$, respectively therefore, $\alpha + \beta = -a$, $\alpha\beta = b$ and $\delta + \gamma = -b$, $\gamma\delta = a$.

Given $|\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha + \beta)^2 - \alpha\beta$

$= (\gamma + \delta)^2 - 4\gamma\delta$

$\Rightarrow a^2 - 4b = b^2 - 4a$

$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$

$\Rightarrow a + b + 4 = 0$ (Because $a \neq b$)

Question 20: If $b_1 b_2 = 2(c_1 + c_2)$, then at least one of the equations $x^2 + b_1x + c1 = 0$ and $x^2 + b_2x + C_2 = 0$ has ____________ roots.

Solution:

Let D1 and D2 be discriminants of $x^2 + b_1x + c1 = 0$ and $x^2 + b_2x + C_2 = 0$, respectively.

Then,

$D_1 + D_2 = b_1^2 - 4c_1 + b_2^2 - 4c_2$

$= (b_1^2 + b_2^2) - 4(c_1 + c_2)$

$= b_1^2 + b_2^2 - 2b_1b_2$ [Because $b_1b_2 = 2(c_1 + c_2)$] $= (b_1 - b_2)^2 \geq 0$

$\Rightarrow D_1 \geq 0$ or $D_2 \geq 0$ or D_1 and D_2 both are positive.

Hence, at least one of the equations has real roots.

Question 21: If the roots of the equation $x^2 + 2ax + b = 0$ are real and distinct and they differ by at most 2m then b lies in what interval?

Solution:

Let the roots be α, β

$\alpha + \beta = -2a$ and $\alpha\beta = b$

Given, $|\alpha - \beta| \leq 2m$

or $|\alpha - \beta|^2 \leq (2m)^2$ or

$(\alpha + \beta)^2 - 4ab \leq 4m^2$ or

$4a^2 - 4b \leq 4m^2$

$\Rightarrow a^2 - m^2 \leq b$ and discriminant $D > 0$ or

$4a^2 - 4b > 0$

$\Rightarrow a^2 - m^2 \leq b$ and $b < a^2$.

Hence, $b \in [a^2 - m^2, a^2)$.

Question 22: If $([1 + i] / [1 - i])^m = 1$, then what is the least integral value of m?

Solution:

$[1 + i] / [1 - i] = ([1 + i] / [1 - i]) \times [1 + i] / [1 + i]$

$= [(1 + i)^2] / [2]$

$= 2i / 2$

$= i$

$([1 + i] / [1 - i])^m = 1$ (as given)
So, the least value of m = 4 {Because $i^4 = 1$}

Question 23: If $(1 - i)\, x + (1 + i)\, y = 1 - 3i$, then (x, y) = ____________.

Solution:

$(1 - i)\, x + (1 + i)\, y = 1 - 3i$

$\Rightarrow (x + y) + i\,(-x + y) = 1 - 3i$

Equating real and imaginary parts, we get $x + y = 1$ and $-x + y = -3$;

So, $x = 2$, $y = -1$.

Thus, the point is (2, −1).

Question 24: $[3 + 2i \sin\theta] / [1 - 2i \sin\theta]$ will be purely imaginary if θ = ____________.

Solution:

$[3 + 2i \sin\theta] / [1 - 2i \sin\theta]$ will be purely imaginary, if the real part vanishes, i.e.,

$[3 - 4 \sin^2 \theta] / [1 + 4 \sin^2\theta] = 0$

$3 - 4 \sin^2 \theta$ (only if θ be real) =

$\sin\theta = \pm\sqrt{3} / 2$

$= \sin(\pm \pi / 3)$

$\theta = n\pi + (-1)^n (\pm \pi / 3)$

$= n\pi \pm \pi / 3$

Question 25: The real values of x and y for which the equation is $(x + iy)(2 - 3i) = 4 + i$ is satisfied, are __________.

Solution:

Equation $(x + iy)(2 - 3i) = 4 + i$

$(2x + 3y) + i(-3x + 2y) = 4 + i$

Equating real and imaginary parts, we get

$2x + 3y = 4$ ……(i)

$-3x + 2y = 1$ ……(ii)

From (i) and (ii), we get

$x = 5 / 13, y = 14 / 13$

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